**P 7239**

M.E. DEGREE EXAMINATION, MAY/JUNE 2006.

First Semester

Structural Engineering

MA 145 — APPLIED MATHEMATICS

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. State the assumptions made while deriving the equation of vibration of a string in the simple form.

2. State the two laws of thermodynamcis used in the derivation of one dimensional heat flow equation.

3. Write down the three mathematically possible solutions of Laplace equation in two dimensions.

4. Give an example for Harmonic function.

5. Write down the Euler–Lagrange differential equation.

6. Write down the Euler's differential equation for the functional

   $I = \int_{x_1}^{x_2} F(x, y, y', y'')\,dx$ stationary.

7. A continuous random variable $X$ has a pdf $f(x) = Kx^2 e^{-x}$ when $x \geq 0$. Find $K$.

8. If $X$ represents the outcome, when a fair die is tossed, find the moment generating function of $X$.

9. Define the term : Multiple partial, correlation coefficient.

10. What is meant by a sufficient estimator?

PART B — ($5 \times 16 = 80$ marks)

11. (i) Find the coefficient of correlation between industrial production and export using the following data and comment on the result.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Product (in crore tons) : | 55 | 56 | 58 | 59 | 60 | 60 | 62 |
| Exports (in crore tons) : | 35 | 38 | 38 | 39 | 44 | 43 | 44 |

(ii) In a partially destroyed laboratory record if an analysis correlation data, the following results only are legible :

Variance of $x = 9$

Regression equations are $8x - 10y + 66 = 0$; $40x - 18y = 214$

(1) What are the mean values of $x$ and $y$

(2) The S.D. of $y$ and

(3) The coefficient of correlation between $x$ and $y$?

12. (a) Using the Laplace transform method, solve the IBVP described as

$$u_{xx} = \frac{1}{c^2}u_{tt} - \cos\omega t\,,\ 0 \le x < \infty\,,\ 0 \le t < \infty$$

$u(0,t) = 0$, $u$ is bounded as $x$ tends to $\infty$

$u_t(x,0) = u(x,0) = 0$.

Or

(b) Solve the heat conduction problem described by

$$K\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t},\ 0 < x < \infty,\ t > 0$$

$u(0,t) = u_0$, $t \ge 0$

$u(x,0) = 0$, $0 < x < \infty$

$u$ and $\frac{\partial u}{\partial x}$ both tend to zero as $x \to \infty$.

13. (a) Solve the following boundary value problem in the half-plane $y > 0$ described by $u_{xx} + u_{yy} = 0$, $-\infty < x < \infty$, $y > 0$

$u(x,0) = f(x)$, $-\infty < x < \infty$

$u$ is bounded as $y \to \infty$;

$u$ and $\frac{\partial u}{\partial x}$ both vanish as $|x| \to \infty$,

Or

(b) Solve the following potential problem in the semi–infinite strip described by

$$u_{xx} + u_{yy} = 0,\ 0 < x < \infty,\ \ 0 < y < a$$

subject to $u(x,0) = f(x)$,

$$u(x,a) = 0$$

$$u(x,y) = 0\,,\ 0 < y < a,\ 0 < x < \infty$$

$\dfrac{\partial u}{\partial x}$ tends to zero as $x \to \infty$.

14. (a) (i) On what curves can the functional $I = \int_0^1 \left[ \left( \frac{dy}{dx} \right)^2 + 12xy \right] dx$ with $y(0) = 0$ and $y(1) = 1$ be extremized?

(ii) Find the extremals of the functional

$V[y(x), z(x)] = \int_0^{\pi/2} (y'^2 + z'^2 + 2yz)\,dx$ given that $y(0) = 0$, $y\left(\frac{\pi}{2}\right) = -1$, $z(0) = 0$, $z\left(\frac{\pi}{2}\right) = 1$.

Or

(b) (i) Determine the extremal of the functional

$I[y(x)] = \int_{-a}^{a} \left\{ \frac{\mu}{2} y''^2 + \rho y \right\} dx$ that satisfies the boundary conditions :

$y(-a) = 0$, $y'(-a) = 0$, $y(a) = 0$, $y'(a) = 0$.

(ii) Write the Ostrogradsky equation for the functional

$$I[z(x,y)] = \iint_D \left[ \left( \frac{\partial z}{\partial x} \right)^2 + \left( \frac{\partial z}{\partial y} \right)^2 \right] dx\, dy .$$

15. (a) (i) If $r_{12}$ and $r_{13}$ are given, show that $r_{23}$ must lie in the range $r_{13} r_{12} \pm (1 - r_{12}^2 - r_{13}^2 + r_{12}^2 r_{13}^2)^{1/2}$.

(ii) Show that the correlation coefficient between the residuals $x_{1.23}$ and $x_{2.13}$ is equal and opposite to that between $x_{1.3}$ and $x_{2.3}$.

Or

(b) (i) If $T$ is an unbiased estimator of $\theta$, show that $T^2$ and $\sqrt{T}$ are biased estimators of $\theta^2$ and $\sqrt{\theta}$ respectively.

(ii) Find the maximum likelihood estimator for the parameter $\lambda$ of Poisson distribution.

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