P 7242

M.E. DEGREE EXAMINATION, MAY/JUNE 2006.

First Semester

Refrigeration and Air Conditioning

MA 148 — APPLIED MATHEMATICS FOR MECHANICAL ENGINEERS

(Common for M.E. Computer Aided Design, M.E. Energy Engineering, M.E. Engineering Design, M.E. Internal combustion Engineering and M.E. Thermal Engineering)

me : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. What is the value of α^2 in $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$.

2. If u and $\frac{\partial u}{\partial x} \to 0$ as $x \to \infty$, find the Fourier cosine transform of $\frac{\partial^2 u}{\partial x^2}$.

3. Prove that if a harmonic function vanighes every where on the boundary then it is identically zero everywhere.

4. Define interior Dirichlet problem for a rectangle.

5. Write down the expanded form of Euler's equation.

6. Write down the Ostrograd sky equation in calculus of variation.

7. Write down the implicit formula to solve diffusion equation.

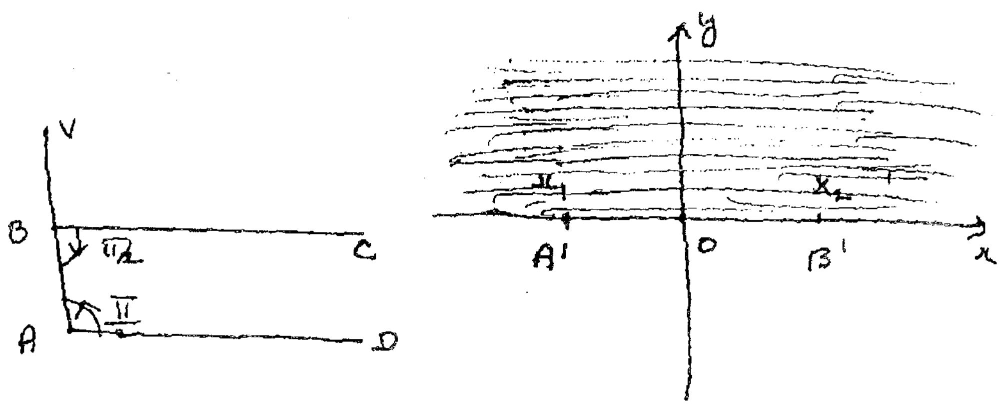
8. In one dimensional heat flow equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$, for what values of $r = \frac{K\alpha^2}{h^2}$, $k = \Delta y$ ad $h = \Delta x$ the explicit method and implicit method are valid.

9. Prove that for closed polygons the sum of the exponents $\frac{\alpha_1}{\pi} - 1, \frac{\alpha_2}{\pi} - 1 \ldots\ldots \frac{\alpha n}{\pi} - 1$ in the Schwartz Christoffel transformation is equal to -2.

10. What is the advantage of choosing one vertex of the polygon at infinity in Schwark Christoffel transformation?

PART B — (5 × 16 = 80 marks)

11. (i) Find the transformation which maps the infinite strip in the w plane shown below in to the upper half of the z plane. (8)



(ii) The complex potential of a fluid flow is given by $w(z) = V_0\left(z + \frac{a^2}{z}\right)$ where V_0 and a are positive constants. Find the equations for the stream lines and equipotential lines. (4)

(iii) Find the transformation which maps the cycloid $x = a(t - \sin t)$ $y = a(1 - \cos t)$ into a straight line. (4)

12. (a) A tightly stretched string has its ends fixed at $x = 0$ and $x = l$. Motion is started by displacing the string into the form $y = k(lx - x^2)$ from which it is released from rest at time $t = 0$. Find the displacement of the string using Laplace transform method. Given that (16)

$$L^{-1}\left[\frac{\cosh sx}{s^2 \cos h\, sa}\right] = \frac{1}{2}\left(t^2 + x^2 + a^2\right) - \frac{16a^2}{\pi^3} \sum_{1}^{\infty} \frac{(-1)^n}{(2n-1)^3} \cos\frac{(2n-1)\pi x}{2a} \cos\frac{(2n-1)\pi t}{2a}$$

Or

(b) Solve the one dimensional heat equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{k}\frac{\partial u}{\partial t}$ for $-\infty \le x \le \infty$, and $t > 0$ subject to the boundary conditions $u = \frac{\partial u}{\partial x} = 0$ as $x \to \pm\infty$ using Fourier transform method. (16)

(a) Using Fourier transform, solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ $x \ge 0$ and $y \ge 0$ subject to the conditions $u(x, y)$ is bounded, $\frac{\partial u}{\partial x}(0, y) = 0$ and $u(x, 0) = x \quad 0 \le x \le 1$
$= 0 \quad x > 1$
(16)

Or

(b) Using Fourier transform, find a function $u(x, y)$ which is harmonic in the open square $0 < x, < \pi$, $0 < y < \pi$ takes a constant value u_0 on the edge $y = \pi$ and vanishes on the edges of the square. (16)

14. (a) (i) Find the stationary function associated with the integral $\int_0^1 y'^2 f(x)\,dx$ when $y(0) = 0$, $y(1) = 1$. (8)

(ii) Find the extremals of the functional

$$\int_{x_0}^{x_1} \left(2yz - 2y^2 + y'^2 - z'^2\right) dx$$ (8)

Or

(b) (i) Prove that the sphere is the solid figure of revolution which for a given surface area has maximum volume (8)

(ii) Find the extremals of the functional

$$\int_{x_0}^{x_1} \left(x^2 y'^2 + 2y^2 + 2xy\right) dx$$ (8)

15. (a) (i) Derive finite difference scheme to solve hyperbolic equation assuming the necessary conditions. (6)

(ii) Solve the Poisson equation

$u_{xx} + u_{yy} = -81\,xy \quad 0 < x < 1,\ 0 < y < 1$ and $u(0, y) = u(x, 0) = 0$

$u(x, 1) = u(1, y) = 100$ in square meshes each of length $h = \frac{1}{3}$. (10)

Or

(b) (i) Using Crank - Nicolson scheme so[illegible] $u_{xx} = 16u_t \quad 0 < x < 1, \quad t > 0$ given that $u(x, 0) = 0, \quad u(0, t) = 0$ a[illegible] $u(1, t) = 100t$. Compute u for one step in t direction taking $h = \frac{1}{4}$.

(ii) Solve $y_{tt} = y_{xx}$ upto $t = 0.5$ with a spacing of 0.1 given th[illegible] $y(0, t) = 0, \quad y(1, t) = 0 \quad y_t(x, 0) = 0, \quad y(x, 0) = 10 + x(1 - x)$
