M.E. DEGREE EXAMINATION, MAY/JUNE 2006.

First Semester

Industrial Engineering

MA 1605 – PROBABILITY AND STATISTICS

(Common to M.E. CAD/CAM and ME–Industrial Safety Engineering)

(Regulation 2005)

Time : Three hours   Maximum : 100 marks

Answer ALL questions.

(Use of Statistical Tables is permitted)

PART A — (10 × 2 = 20 marks)

1. If at least one child in a family with 2 children is a boy, what is the probability that both children are boys?

2. Given the probability function

x	0	1	2	3
$p(x)$	·1	·3	·5	·1

   Find mean of $y = x^2 + 2x$.

3. Define unbiasedness.

4. State any two properties of multiple correlation coefficient.

5. 900 coins are tossed, head appears 460 times. Compute the test statistic.

6. Define standard error.

7. What is the main aim of the design of experiments?

8. What is the main advantage of LSD over RBD?

9. Write any two properties of moving average method.

10. What is meant by seasonal variation?

PART B — (5 × 16 = 80 marks)

11. (i) Assuming that trend is absent calculate the seasonal indices.

Year	I Quarter	II Quarter	III Quarter	IV Quarter
1981	3.7	4.1	3.3	3.5
1982	3.7	3.9	3.6	3.6
1983	4.0	4.1	3.3	3.1
1984	3.3	4.4	4.0	4.0

(ii) Population figures for a city are given below. Fit $y = ab^x$ and estimate population for 1977.

Year (x) :		1971	1972	1973	1974	1975
Population in thousands (y) :		132	142	157	170	191

12. (a) (i) Find the mgf of RVX having the pdf

$$f(x) = x \quad (0, 1)$$
$$= 2 - x \quad (1, 2)$$
$$= 0 \quad \text{otherwise.}$$

(ii) The joint density function of x and y is

$$f(x,y) = e^{-(x+y)} \quad x \ge 0, \; y \ge 0$$
$$= 0 \quad \text{otherwise.}$$

Find $p(x < 1)$ and $p(x + y < 1)$.

Or

(b) (i) The time in hours required to repair a machine is exponentially distributed with parameter $\lambda = \dfrac{1}{2}$. What is the probability that the repair time exceeds 2 hours.

Also find the conditional probability that a repair takes at least 10 hours given that its duration exceeds 9 hours.

(ii) The two regression lines are $8x - 10y + 66 = 0$, $40x - 18y - 214 = 0$. Find $\bar{x}, \bar{y}, r$.

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(i) Find $R_{1.23}$, $r_{12.3}$ given

$$r_{12} = \cdot 7 \qquad r_{23} = \cdot 5 \qquad r_{31} = \cdot 5$$

(8)

(ii) $\{x_1 \ x_2 \ x_3\}$ is a random sample from a normal population with mean μ and variance σ^2

Test whether the estimators $\hat{\theta}_1$ and $\hat{\theta}_2$ are unbiased.

$$\hat{\theta}_1 = \frac{x_1 + x_2 + x_3}{3} \qquad \hat{\theta}_2 = \frac{x_1 + 2x_2 + 3x_3}{6}$$

Also compare their efficiencies.

(8)

Or

(b) (i) For random sampling from normal population $N\left(\mu, \sigma^2\right)$ find the maximum likelyhood estimation for μ, when σ^2 is known. (8)

(ii) Find Regression equation of x_1 on x_2 and x_3 given

$$\bar{x}_1 = 28.02 \qquad \bar{x}_2 = 4.91 \qquad \bar{x}_3 = 594$$

$$\sigma_1 = 4.42 \qquad \sigma_2 = 1.1 \qquad \sigma_3 = 85$$

$$r_{12} = \cdot 8 \qquad r_{23} = - \cdot 56 \qquad r_{31} = - \cdot 4$$

(8)

(a) (i) Test whether difference between means and difference between S.D's significant.

$$n_1 = 1000 \qquad \bar{x}_1 = 67.42 \qquad S.D = S_1 = 2.58$$

$$n_2 = 1200 \qquad \bar{x}_2 = 67.25 \qquad S_2 = 2.5$$

Use 1% level.

(8)

(ii) A random sample of 10 boys had the following IQ's. 70, 120, 110, 101, 88, 83, 95, 98, 107, 100. Do these data support the assumption of a population mean IQ of 100? Use 5% level.

(8)

Or

(b) (i) The following figures shows the distribution of digits in numbers chosen at random from a telephone directory. Test whether digits occur equally frequently.

Digits	0	1	2	3	4	5	6	7	8	9
Frequency	1026	1107	997	966	1075	933	1107	972	964	853

Use 5% level.

(8)

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(ii) From the following table test whether the treatments are sam[e] respect to yields in tons.

			Yields		
Treatments	A	5	7	3	1
	B	4	4	7	
	C	3	5	1	

Use 5% level.

15. (a) The following data represent the numbers of units of production per turned out by 4 different workers by 5 different machines. Test whe[ther] mean productivity is same

   (i) w.r.t. workers

   (ii) w.r.t. machines

Workers	Machines				
	A	B	C	D	E
1	4	5	3	7	6
2	6	8	6	5	4
3	7	6	7	8	8
4	3	5	4	8	2

Or

(b) Analyse the data in LSD

20 B	17 C	25 D	34 A
25 A	21 D	15 C	24 B
24 D	26 A	21 B	19 C
26 C	23 B	27 A	22 D

Use 5% level of significance.

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