Q 8227

M.E. DEGREE EXAMINATION, MAY/JUNE 2006.

First Semester

Power System Engineering

MA 1614 — APPLIED MATHEMATICS FOR ELECTRICAL ENGINEERS

(Common to M.E. – Power Electronics and Drives, M.E. – Control and Instrumentation and M.E. – High voltage Engineering)

(Regulation 2005)

Time : Three hours   Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the generalised eigen vector of rank 3 corresponding to the eigen value $\lambda = 7$ for the matrix $A = \begin{pmatrix} 7 & 1 & 2 \\ 0 & 7 & 1 \\ 0 & 0 & 7 \end{pmatrix}$.

2. Define singular value of a matrix A.

3. Verify whether the functional $\int_0^{2\pi} (y'^2 - y^2)\,dx$ subject to $y(0) = 1$, $y(2\pi) = 1$ has unique solution.

4. What do you mean by Geodesics in calculus of variation?

5. Find all basic solutions of the L.P.P :

   Max $z = x_1 + x_2 + x_3$ subject to $x_1 + 2x_2 + x_3 = 4$, $2x_1 + x_2 + 5x_3 = 5$, $x_1, x_2, x_3 \geq 0$.

6. When the problem of degeneracy arises in the transportation problem?

7. State any four applications of Dynamic programming.

8. State any four characteristics of dynamic programming.

9. Define linear time invariant system.

10. A stationary random process $X(t)$ has an auto correlation function given by $R_{XX}(Z) = \dfrac{25Z^2 + 36}{6.25Z^2 + 4}$. Find the variance of $X(t)$.

PART B — (5 × 16 = 80 marks)

11. (i) Construct a singular value decomposition for the matrix $A = \begin{pmatrix} 2 & 2 & -2 \\ 2 & 2 & -2 \\ -2 & -2 & 6 \end{pmatrix}$. (8)

(ii) Construct a QR decomposition for the matrix

$$A = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}.$$ (8)

12. (a) (i) Find the extremals of the functional $\int_0^{\pi/2} (y''^2 - y^2 + x^2)\,dx$ that satisfies the conditions $y(0) = 1$, $y'(0) = 0$, $y\left(\frac{\pi}{2}\right) = 0$ and $y'\left(\frac{\pi}{2}\right) = -1$. (8)

(ii) Find the shortest distance between the point A(1, 0) and the ellipse $4x^2 + 9y^2 = 36$. (8)

Or

(b) (i) Find the extremals of the functionals $\int_1^2 \frac{\sqrt{1 + y'^2}}{x}\,dx$, subject to $y(1) = 0$, $y(2) = 1$. (8)

(ii) Prove that the sphere is the solid figure of revolution which for a given surface area has maximum volume. (8)

13. (a) (i) A company manufactures 2 types of printed circuits. The requirements of transistors, resistors and capacitors for each type of printed circuits along with other data are given below : (8)

	Circuit A	Circuit B	Stock available
Transistor	15	10	180
Resistor	10	20	200
Capacitor	15	20	210
Profit	Rs. 5	Rs. 8	

How many circuits of each type should the company produce from the stock to earn maximum profit? Solve graphically.

(ii) A company has four Machines to do three jobs. Each job can be assigned to one and only one machine. The cost of each job on each machine is given in the following table.

Jobs \ Machines	1	2	3	4
A	18	24	28	32
B	8	13	17	19
C	10	15	19	22

What are job assignments which will minimize the cost? (8)

Or

(b) (i) Using simplex method, solve the LPP to Maximize $z = 6x_1 + 4x_2$ subject to the constraints $2x_1 + 3x_2 \le 30$, $3x_1 + 2x_2 \le 24$, $x_1 + x_2 \ge 3$ and $x_1, x_2 \ge 0$. (8)

(ii) Solve the following transportation problem to maximize profit. (8)

Profit (Rs.)/Unit

Source	A	B	C	D	Supply
1	40	25	22	33	100
2	44	35	30	30	30
3	38	38	28	30	70
Demand	40	20	60	30	

14. (a) Use dynamic programming to solve

Max $Z = y_1 y_2 y_3$ subject to the constraints $y_1 + y_2 + y_3 = 5$, $y_1, y_2, y_3 \ge 0$ (16)

Or

(b) Solve the following LPP using dynamic programming.

Max $z = 3x_1 + 5x_2$ subject to

$x_1 \le 4$

$x_2 \le 6$

$3x_1 + 2x_2 \le 18$ and $x_1, x_2 \ge 0$. (16)

15. (a) (i) Given a random variable y with characteristic function $\phi(\omega) = E\left[e^{i\omega y}\right]$ and a random process defined by $x(t) = \cos(\lambda t + y)$. Show that $\{ x(t) \}$ is stationary in the wide sense if $\phi(1) = \phi(2) = 0$. (8)

(ii) Find the power density spectrum of the random process $x(t) = A\cos(\omega_0 t + \theta)$ where A and ω_0 are constant θ is uniformly distributed over $(0, 2\pi)$. (8)

Or

(b) (i) Consider two random process $x(t) = 3\cos(\omega t + \theta)$ and $y(t) = 2\cos(\omega t + \phi)$ where $\phi = \theta - \frac{\pi}{2}$ and θ is uniformly distributed random variable over $(0, 2\pi)$. Verity that

$$|R_{XX}(Z)| \le \sqrt{R_{XX}(0)\ R_{YY}(0)}$$ (8)

(ii) If $x(t)$ is a Gaussian process with $\mu(t) = 10$ and $C(t_1, t_2) = 16\,\bar{e}^{|t_1 - t_2|}$ find the probability that (1) $x(10) \le 8$ (2) $|x(10) - x(6)| \le 4$. (8)
