Y 3001

M.C.A. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2006.

Elective

CA 031 — NUMERICAL AND STATISTICAL METHODS

(Regulation 2002)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. What is the principle in Gauss elimination method?

2. Iteration method will coverage always or not.

3. Given $y_3 = 2$, $y_4 = -6$, $y_5 = 8$, $y_6 = 9$, $y_7 = 17$. Find $\Delta^4 y_3$.

4. Give the order of error in Trapezoidal rule.

5. How many basic values are required for Milnes predictor-corrector method.

6. What are the disadvantages of Taylors series method?

7. Identify the distribution with the following moment generating function $M_x(t) = e^{\frac{t^2}{2}}$.

8. Define hazard function.

9. The joint density function of the random variable X and Y is $f(x,y) = 2$, $0 < x < y < 1$. Find the marginal density function of X.

10. Define absorbing state in a Markov chain.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Solve the following system by Gauss Jordan method.

$$2x + y + 3z = 13$$
$$x + 5y + z = 14$$
$$3x + y + 4z = 17$$

(ii) Solve the following equations of Gauss Seidel method.

$$-2x + 3y + 10z = 22$$
$$x + 10y - z = -22$$
$$10x + 2y + z = 9$$

Or

(b) (i) Solve the following system by Gauss elimination method.

$x + 2y + z = 3$, $2x + 3y + 3z = 10$, $3x - y + 2z = 13$

(ii) Solve the following system by Gauss-Jacobic iteration method.

$x + y + 54z = 110$, $27x + 6y - z = 85$, $6x + 15y + 2z = 72$

12. (a) (i) Find the polynomial satisfying the data :

x :	1	2	– 4
$f(x)$:	3	– 5	4

(ii) Find the first and second derivative at $x = 16$ from the following data :

x :	15	17	19	21	23	25
y :	3.873	4.123	4.359	4.583	4.796	5

Or

(b) (i) Using Simpson's rule evaluate $\int_0^{\frac{\pi}{2}} e^{\sin x} dx$.

(ii) By Gaussian three point quadrature formula. Find $\int_2^3 \frac{dt}{1+t}$.

13. (a) (i) By Taylors series method find $y(0.1)$ and $y(0.2)$ given that $y'' = y + xy'$, $y(0) = 1$, $y'(0) = 0$.

(ii) Given $\frac{dy}{dx} = x - y^2$, $y(0) = 0$, $y(0.2) = 0.02$ $y(0.4) = 0.0795$, $y(0.6) = 0.1762$ using Milnes predictor-corrector method find $y(0.8)$.

Or

(b) (i) Using Runge Kutta method find $y(0.1)$ given that $y' = \frac{1}{2}(1+x)y^2$, $y(0) = 1$.

(ii) $\frac{dy}{dx} = xy + y^2$, $y(0) = 1$, $y(0.1) = 1.1169$, $y(0.2) = 1.2774$, $y(0.3) = 1.5041$. Find $y(0.4)$ using Adam's method.

14. (a) (i) Find the moment generating function for the binomial random variable and hence find its mean and variance.

(ii) For the exponential failure distribution find $R(t)$, $Z(t)$ and $E(t)$.

Or

(b) (i) Find the characteristic function for the normal distribution.

(ii) The life time in hours of a component is a random variable X which follows Weibull distribution with $\alpha = 0.1$, $\beta = 0.5$. Find the mean life time of the component and also the probability that such component will have more than 300 hours.

15. (a) (i) The joint p.d.f of the random variable X and Y is given by $f(x,y) = \begin{cases} 6(1-x-y), & x > 0, y > 0 \\ & x + y < 1 \\ 0, & \text{otherwise} \end{cases}$. Find the marginal and conditional distribution of X and Y.

(ii) Show that the random process $X(t) = A\cos wt \times B\cos wt$ where A and B are independent random variables with zero means and equal variances in stationary in wide sense.

Or

(b) (i) The joint probability distribution of X and Y is given by $f(x,y) = \frac{x+y}{21}$, $x = 1, 2, 3$, $y = 1, 2$. Find the marginal distributions of X and Y.

(ii) The transition probability matrix of a Markov chain $\{X_n\}$, $n = 1, 2, \cdots$ having three states 1, 2, and 3 is $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$

and the initial distribution is $p^{(0)} = (0.7, 0.2, 0.1)$.

Find :

(1) $p(X_2 = 3)$

(2) $p(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$.