Y 3037

M.C.A. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2006.

First Semester

MA 154 — MATHEMATICAL FOUNDATION OF COMPUTER SCIENCE

(Regulation 2002)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Show that $P \rightarrow (Q \rightarrow R) \Leftrightarrow (P \wedge Q) \rightarrow R$, without using truth table.

2. Prove that $(\forall x)(P(x) \rightarrow M(x)), P(s) \Rightarrow M(s)$.

3. How many 4 digit numbers, not beginning with zero, without repeating any digit can be formed using 0, 1, 2, 3, 4.

4. State pigeon hole principle.

5. Prove that a group and its subgroup will have the same identity element.

6. Give two examples for ring.

7. Express absolute function in terms of proper subtraction function.

8. Define regular function.

9. State the distributive inequalities of lattice.

10. In a Boolean algebra if $b' \leq a'$, prove that $a \leq b$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the PDNF of negation of $P \vee (\neg P \wedge \neg Q \wedge R)$. (8)

(ii) Using C.P rule, prove that $R \rightarrow S$ can be derived from $P \rightarrow (Q \rightarrow S), \neg R \vee P, Q$. (8)

Or

(b) (i) Prove that $P \to Q, Q \to R, S \to \neg R, P \wedge S$ are inconsistent premises. (8)

(ii) What is indirect method in Predicate Calculus? Use it to prove $(\forall x)(P(x) \to Q(x)), (\exists x) P(x) \Rightarrow (\exists x) Q(x)$. (8)

12. (a) (i) State the principle of mathematical induction. Use it to prove $a + ar + ar^2 + \cdots + ar^n = \dfrac{ar^{n+1} - a}{r - 1}$. (8)

(ii) Using G.F., solve $a_n - 8a_{n-1} = 10^{n-1}, a_1 = 9$. (8)

Or

(b) (i) Determine the number of positive integers n, $1 \le n \le 2000$ that are divisible by 2, 3, 5 or 7. (8)

(ii) Find the general solution of $a_n - 6a_{n-1} + 9a_{n-2} = n2^n$. (8)

13. (a) (i) Prove that every subgroup of a cyclic group is cyclic. (8)

(ii) Define monoid and submonoid. Prove that the set of idempotent elements of an abelian monoid will form a submonoid. (8)

Or

(b) (i) Prove that every group of order n will be isomorphic to a permutation group of degree n. (8)

(ii) Given the multiplicative group $G = \{1, -1, i, -i\}$ and $H = \{1, -1\}$ is its subgroup. Find the left and right cosets of H. What is the index of H? (8)

14. (a) (i) Define successor function and projection function. If $f(x, y) = x + y$, express $f(x, y+1)$ in terms of these two functions. (8)

(ii) Define partial recursive function. Show that the function $f(x) = x/2$ is partial recursive. (8)

Or

(b) (i) Show that the product function is primitive recursive. (8)

(ii) Define Ackermann's function $A(x, y)$. Find $A(1, 4)$. (8)

15. (a) (i) Prove that $(\{1,2,3,5,30\}, \leq)$ where $\leq$ is divisibility relation is a poset. Prove this poset is lattice. (10)

(ii) State and prove the Demorgan laws of Boolean Algebra. (6)

Or

(b) (i) In a lattice $(L, \leq, \vee, \wedge)$, prove that the following two statements are equivalent :

(1) $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

(2) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$ where $a, b, c \in L$. (8)

(ii) Prove that in a distributive complemented lattice, the complement of an element is unique. (8)
