Z 3506

M.C.A. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2006.

Second Semester

MC 1651 — MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE

(Regulation 2005)

Time : Three hours   Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Can the system of equations $AX = B$ with the following augmented matrix have unique solution?

$$(A/B) = \begin{bmatrix} 2 & 4 & 1 & 2 \\ 0 & 3 & -1 & 1 \\ 0 & 0 & 0 & k \end{bmatrix}$$

2. If -2, 3, 6 are the eigen values of a 3 × 3 square matrix A, then what are the eigen values of $6 \cdot A^{-1}$?

3. Can a relation which is irreflexive and symmetric be transitive? Why or why not?

4. Let f and g be two functions. When is the composite function $g \circ f$ is defined?

5. Write the negation of the statement below :

   'If there is a will, then there is a way'

6. Obtain the Disjunctive Normal Form (DNF) for the formula $(p \wedge (q \to r)) \to r$.

7. What is the role of Context-free grammars in programming languages?

8. Show that the grammar with productions $S \to aAb / abSb / a$, $A \to bS / aAAb$ is ambiguous by constructing two derivation trees for the word $ab\,ab$.

9. How a Non-deterministic Finite-state Automaton (NFA) differs from a Deterministic Finite-state Automaton (DFA)?

10. Draw the state diagram of the automaton $M = (\{s_0, s_1, s_2\}, \{0,1\}, \delta, \{s_0, s_2\})$ represented by the state table below.

	δ	Input 0	Input 1
State	s_0	s_1	s_2
	s_1	s_1	s_2
	s_2	s_1	s_2

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the rank of the matrix (6)

$$\begin{pmatrix} 1 & -1 & 2 & 1 \\ 3 & 1 & 1 & 4 \\ 1 & 3 & -3 & 2 \\ 5 & -1 & 5 & 6 \end{pmatrix}$$

(ii) Find the eigen values and eigen vectors of the matrix (10)

$$\begin{pmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{pmatrix}$$

Or

(b) (i) Check whether the following system of equations is consistent or not. If so, find the solution. (6)

$$x + y + 2z = 4$$
$$2x - y + 3z = 9$$
$$3x - y - z = 2.$$

(ii) Obtain the inverse of the matrix A using Cayley Hamilton theorem. (10)

$$A = \begin{pmatrix} 3 & -4 & 2 \\ -2 & 1 & 0 \\ -1 & -1 & 1 \end{pmatrix}$$

12. (a) (i) Which of the following functions are one-one, onto and bijections? Find the inverse for any function if exists. (10)

(1) $f_1 : R \to R$ defined by $f_1(x) = x^3 - x$.

(2) $f_2 : Z \to Z$ defined by $f_2(x) = |x|$.

(3) $f_3 : Z \to Z$ defined by $f_3(x) = x^3$.

(4) $f_4 : R \to R$ defined by $f_4(x) = 2x + 1$.

(ii) In how many ways we can arrange the seven letters of the word SUCCESS? Among these arrangements, how many of them have all the three S appear consecutively? (6)

Or

(b) (i) Out of 100 sportsmen in a college, 39 play Tennis, 58 play Cricket and 32 play Hockey. 10 play Cricket and Hockey, 11 play Hockey and Tennis, and 13 play Tennis and Cricket.

How many play (1) All the three games

(2) Just one game

(3) Tennis and Cricket but not hockey. (8)

(ii) Prove that the relation defined by '$a\,R\,b$ if and only if a-b is divisible by m' (congruence modulo m) on the set of integers is an equivalence relation. (8)

13. (a) (i) Test the validity of the following argument

Sania is watching TV

If Sania is watching TV, then she is not studying

If she is not studying, then her father will not buy her a scooty

Therefore Sania's father will not buy her a scooty. (10)

(ii) Translate the following predicate calculus formula into English sentence

$\forall x\,[C(x) \vee \exists y(C(y) \wedge F(x, y))]$

Here, $C(x)$: x has a computer, $F(x, y)$: x and y are friends. The universe for both x and y is the set of all students of your college. (6)

Or

(b) (i) Show that $\neg p \to (q \to r)$ and $q \to (p \vee r)$ are logically equivalent. (10)

(ii) Express the following two statements symbolically using quantifiers.

(1) Some students in this examination hall know java.

(2) Every student in this examination hall knows C++ or Java. (6)

14. (a) (i) Consider the grammar $G = (N, T, S, P)$ where $N = \{S, B\}$, $T = \{a, b, c\}$ and $P = \{S \to aS\,Bc,\ S \to abc,\ cB \to Bc,\ bB \to bb\}$. Check whether this grammar generates the words – *abc, aabbcc, aab, aaabbbccc, abbccc*. Hence identify the language generated by this grammar. (10)

(ii) State the *Pumping lemma* for regular languages. How do you apply *Pumping lemma* to prove that certain languages are not regular? (6)

Or

(b) (i) Classify the Phrase-structure grammars according to the types of production rules that are allowed. Also write the Chomsky Hierarchy exhibiting the relationship between the four types of languages generated by these grammars. (10)

(ii) Construct a context-free grammar for the language $L = \{wcw^R : w \in \{a, b\}^*\}$. Here w^R denotes the reverse of the string w. (6)

15. (a) (i) Construct a DFA equivalent to the NFA represented by the state-diagram in Figure 1. (10)

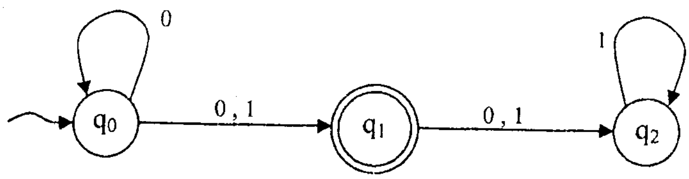

Figure 1

(ii) Which of the following strings are recognized by the DFA in Figure 2.

(1) 010 (2) 1101 (3) 1111110 (4) 010101010

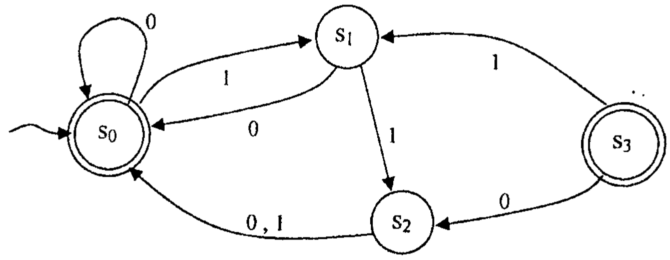

Figure 2. (6)

Or

(b) (i) Construct a Non-deterministic Finite-state Automaton that recognizes the regular set $1^* \cup 01$. (10)

(ii) Determine the language accepted by the Non-deterministic Finite-state Automaton in Figure 3. (6)

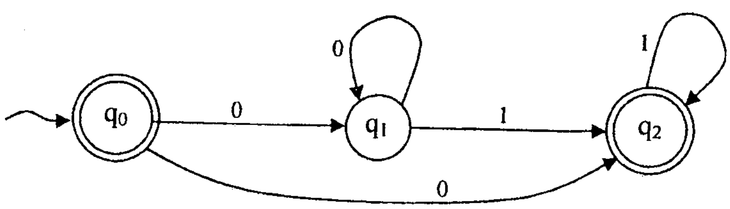

Figure 3.
