Reg. No. : ☐☐☐☐☐☐☐☐☐☐☐☐☐

K 3529

M.C.A. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2007.

Second Semester

MC 1651 — MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE

(Regulation 2005)

Time : Three hours   Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then find AA^T.

2. Find the eigenvalues of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.

3. If $A = \{a, b\}$, $B = \{1, 2\}$, compute $A \times B$.

4. Let $A = \{1, 2, 3, 4\}$ and r be the relation $\leq$ on A. Draw the Hasse diagram.

5. Prove that $(p \wedge q) \Rightarrow (p \rightarrow q)$.

6. Is $\{\uparrow\}$ functionally complete?

7. Give the context free grammar for generating the language $\{ww^R / w \in \{a, b\}^*\}$.

8. Is $\{b^{2n} : n \geq 1\}$ a regular? Justify the claim.

9. Construct a automata to accept the string $a(bb)^*ba$.

10. , find the language generated by the above automata .

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the non-trivial solution of the equation $x - y + 2z - 3w = 0$, $3x + 2y - 4z + w = 0$, $5x - 3y + 2z + 6w = 0$, $x - 9y + 14z - 2w = 0$. (8)

   (ii) Find the eigenvalues and the eigenvectors of the matrix

   $$A = \begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}.$$ (8)

Or

(b) (i) Verify the Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.

   (ii) and hence find A^4 and A^{-1}. (8 + 8)

12. (a) (i) Among 50 students in a class, 26 passed in first semester and 21 passed in second semester examinations. If 17 did not pass in either semester, how many passed in both semesters? (8)

   (ii) If A, B, C are any three sets, then prove that

   $$A \times (B \cup C) = (A \times B) \cup (A \times C)$$ (8)

Or

(b) (i) Prove that the relation "congruence modulo M, over the set of positive integers is an equivalence relation. (8)

   (ii) If $f : A \to B, g : B \to C$ and $h : C \to D$ then prove that $(h \circ g) \circ f = h \circ (g \circ f)$. (8)

13. (a) (i) Find the principal conjunctive normal form and the disjunctive normal form of $p \vee (\neg p \to (q \vee (\neg q \to r)))$. (8)

   (ii) Test the consistency of the following statement :

   The contract is satisfied iff the building is completed by November 30. The building is completed by November 30 iff the electrical subcontractor completes his work by November 10. The bank loses money iff the contract is not satisfied. The electrical sub contractor completes his work by November 10 iff the bank loses money. (8)

Or

2

K 3529

(b) (i) Show that $(\exists x)M(x)$ follows logically the premises $(\forall x)(H(x) \rightarrow M(x)) \wedge (\exists x)(H(x))$. (8)

(ii) Prove by indirect method

$\neg q, p \rightarrow q, p \vee r \Rightarrow r$. (8)

14. (a) (i) Construct a grammar G for the language $L(G) = \{a^n b a^m : n, m \geq 1\}$. (8)

(ii) State and prove the pumping lemma. (8)

Or

(b) (i) Write the language generated by $G = (\{S, B, C\}, \{a, b, c\}, P, S)$ where P consists of

$S \rightarrow aSBC, S \rightarrow aBC, CB \rightarrow BC, aB \rightarrow ab$.

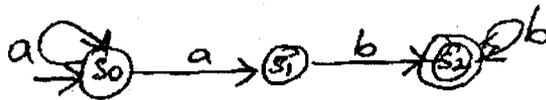$bB \rightarrow bb, bC \rightarrow bc, cC \rightarrow cc$. (8)

(ii) Show that $L = \{a^n b^n / n \geq 1\}$ is not regular. (8)

15. (a) (i) Construct a finite automata $L = \{aw*a/w* \in \{a, b\}\}$ (8)

(ii) Prove that if L is accepted by NFA, then there exists a DFA to accept L.

(8)

Or

(b) (i) Construct a DFA for the given NFA. (8)

(ii) If L is accepted by a DFA, then show that L is denoted by a regular expression. (8)