Reg. No. :

K 3506

M.C.A. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2007.

Elective

MC 1621 — NUMERICAL AND STATISTICAL METHODS

(Regulation 2005)

Time : Three hours | Maximum : 100 marks

(Statistical tables are permitted)

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Compare Gaussian elimination and Gauss Jordan methods in solving linear system $Ax = B$.

2. Write a sufficient condition for convergence of Gauss-Seidel method.

3. Find $f(2)$ by Lagrange's interpolation method using the data :

x :	–1	1	3
$f(x)$:	2	0	4

4. Evaluate $\int_{-1}^{1} \frac{dx}{1+x^2}$ using 2 point Gaussian formula.

5. Given $y' = x^2 + y^2, y(0) = 1$, compute $y(0.1)$ by Euler's method, where $y' = \frac{dy}{dx}$.

6. What is a predictor-corrector method for solving a differential equation?

7. Determine the value of K for which the function given by $f(x, y) = k\,(x + y)$ for $x = 1, 2, 3$ and $y = 1, 2, 3$ can be serve as a joint probability distribution.

8. The probability that A will live upto 60 years is $\frac{3}{4}$ and the probability that B will live upto 60 years is $\frac{2}{3}$. What is the probability

   (a) that both A and B will live upto 60 years

   (b) that both will die before reaching 60 years.

9. A sample of size 13 gave an estimated population variance of 30, while another sample of size 15 gave an estimate of 2.5. Tabulated value of F at 5% level of significance for $v_1 = 12$ and $v_2 = 14$ degrees of freedom is 2.53. Could both samples be from populations with the same variance?

10. What is the use of χ^2 (Chi-square) test?

PART B — (5 × 16 = 80 marks)

11. (a) (i) Solve the following system of equations by Gauss-elimination method : (8)

$$x + 2y + z = 3$$
$$2x + 3y + 3z = 10$$
$$3x - y + 2z = 13$$

(ii) Solve :

$$8x - 3y + 2z = 20$$
$$4x + 11y - z = 33$$
$$6x + 3y + 12z = 35,$$

using Gauss-Seidel method (perform 4 iterations). (8)

Or

(b) (i) Solve the following system of equations by Gauss-Jordan method. (8)

$$x + y + z = 9$$
$$2x - 3y + 4z = 13$$
$$3x + 4y + 5z = 40$$

(ii) Solve the following system of equations by Gauss Jacobi method (perform 5 iterations) (8)

$$10x - 5y - 2z = 3$$
$$4x - 10y + 3z = -3$$
$$x + 6y + 10z = -3$$

12. (a) (i) Find $\frac{dy}{dx}$ and $\left(\frac{d^2y}{dx^2}\right)$ at 1.6 for the following data : (8)

x :	1.0	1.1	1.2	1.3	1.4	1.5	1.6
y :	7.989	8.403	8.781	9.129	9.451	9.750	10.031

(ii) Evaluate the integral $\int_4^{5.2} \log_e^x \, dx$ using Simpson's $\frac{1}{3}$ and $\frac{3}{8}$ rule. (8)

Or

(b) (i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ of $(x)^{1/3}$ at $x = 50$ using Newtons forward differential formula for the following data : (8)

x :	50	51	52	53	54	55	56
$y = (x)^{1/3}$:	3.684	3.7084	3.7325	3.7563	3.7798	3.8030	3.8259

(ii) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Trapezoidal rule with $h = 0.2$ and compare with actual integration, hence obtain the approximate value of π. Can you use Simpson's formulae in this case? (8)

13. (a) (i) Using Taylor's series method find the value of y at $x = 0.1$ and $x = 0.2$ to five places of decimals if $\frac{dy}{dx} = x^2y - 1$, given $y(0) = 1$. (8)

(ii) Given the first order differential equation $\frac{dy}{dx} = x^2(1+y)$ and $y(1) = 1, y(1.1) = 1.233\ y(1.2) = 1.548, y(1.3) = 1.979$, evaluate $y(1.4)$ by Adam's Bashforth predictor-corrector method. (8)

Or

(b) (i) Apply Runge-Kutta fourth order method to find the value of y when $x = 0.4$ given that $\frac{dy}{dx} = x + y$ and $y = 1$ when $x = 0$, (taking $h = 0.2$) (8)

(ii) Determine the value of $y(0.4)$ using Milne's predictor-corrector method, given $y' = xy + x^2$, $y(0) = 1$, $y(0.1) = 1.1167$, $y(0.2) = 1.2767$, $y(0.3) = 1.5023$. (8)

14. (a) (i) A continuous random variable X has probability density

$$f(x) = \begin{cases} Ke^{-3x} & \text{for } x > 0 \\ 0 & \text{elsewhere} \end{cases}$$

Find

(1) K

(2) Mean

(3) $P(0.5 \le x \le 1)$. (8)

(ii) State Baye's theorem. In a certain state 25 percent of all cars emit excessive amounts of pollutants. If the probability is 0.99 that a car emit excessive amount of pollutant will fail state's vechicular emission test, and the probability 0.17 that a car not emitting excessive amounts of pollutants will nevertheless fail the test. What is the probability that a car that fails the test actually emits excessive amounts of pollutants? Use Baye's theorem. (8)

Or

(b) (i) The joint probability density function of the two dimensional random variable (X, Y) is given by $f(x, y) = \begin{cases} \frac{8}{9}xy & 1 \leq x \leq y \leq 2 \\ 0 & \text{elsewhere} \end{cases}$

(1) Find the marginal density function of X and Y.

(2) Find the conditional density of Y given $X = x$. (10)

(ii) If X represents outcome, when a fair die is tossed, find the moment generating function of X and hence find mean and variance of X. (6)

15. (a) (i) The mean life time of a sample of 25 bulbs is found as 1550 h, with a standard deviation of 120 h. The company manufacturing the bulbs claims that the average life of their bulbs is 1600 h. Is the claim acceptable at 5% level of significance? (6)

(ii) The following data give the number of aircraft accidents that occurred during the various days of a week.

Day :	Mon	Tue	Wed	Thu	Fri	Sat
No. of accident :	15	19	13	12	16	15

Test whether the accidents are uniformly distributed over the week. Use 5% level of significance. (10)

Or

(b) Three salesman A, B, C were posted in different areas by a company. The number of units of commodity X sold by them are as follows :

A	B	C
6	5	5
7	5	4
3	3	3
8	7	4

On the basis of this information, can it be concluded that there is a significant difference in the performance of the three salesman? Use 5% level of significance. (16)