B 477

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2005.

Fourth Semester

Computer Science Engineering

MA 040 — PROBABILITY AND QUEUEING THEORY

Time : Three hours | Maximum : 100 marks

Statistical tables are permitted.

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Let $P(A \cup B) = 5/6$, $P(A \cap B) = \frac{1}{3}$ and $P(\overline{B}) = \frac{1}{2}$. Are the events A and B independent. Explain.

2. A continuous random variable X has the p.d.f. $f(x)$. given by $f(x) = \{C\, e^{-|x|} \quad -\infty < x < \infty$. Find the value of C and C.D.F. of X.

3. Prove that the correlation coefficient ρ_{XY} takes value in the range –1 to 1.

4. If the joint p.d.f. of (X, Y) is $f(x, y) = \begin{cases} \frac{1}{4}, & 0 \le x, y \le 2 \\ 0, & \text{otherwise.} \end{cases}$

   find $P(x + y \le 1)$.

5. Distinguish between wide sense stationary and strict sense stationary random processes.

6. Obtain the moment generating function for the Poisson process.

7. Find the invariant probabilities for the Markov Chain $\{X_n \; ; n \ge 1\}$ with state space $S = \{0, 1\}$ and one–step TPM $P = \begin{bmatrix} 1 & 0 \\ 1/2 & 1/2 \end{bmatrix}$.

8. A component has MTBF = 100 hours and MTTR = 20 hours with both failure and repair distributions exponential. Find the availability and non–availability of the component after a long time.

9. In a given M/M/1/∞/FCFS queue, ρ = 0.6, what is the probability that the queue contains 5 or more customers?

10. What is the effective arrival rate for M/M/1/4/FCFS queueing model when $\lambda = 2$ and $\mu = 5$.

PART B — (5 × 16 = 80 marks)

11. (i) Let $\{X(t)\,;\,t \geq 0\}$ be a Poisson process with parameter λ. Suppose each arrival is registered with probability 'p' independent of other arrivals. Let $\{X(t)\,;\,t \geq 0\}$ be the process of registered arrivals. Prove that $Y(t)$ is a Poisson process with parameter $\lambda\, P$.

(ii) Let $\{N(t)\,;\,t \geq 0\}$ be a renewal process with distribution $F(t)$. S.T. $P\{\,N(t) = n\} = F^{(n)}(t) - F^{(n+1)}(t)$ and $E^{(N(t))} = \sum_{n=1}^{\alpha_1} F^{(n)}(t)$, where $F^{(n)}(t)$ is the n–fold convolution of $F(t)$ with itself.

(iii) Express the answers to the following questions in terms of probability functions :

(1) State the definition of a Markov random process

(2) State the definition of an independent–increment random process.

12. (a) (i) A box contains 5 red and 4 white balls. Two balls are drawn successively from the box without replacement and it is noted that the second one is white. What is the probability that the first is also white?

(ii) Let X be an exponential distributed R.V. with parameter $\lambda = 1$. Use Chebyschev inequality to show that $p(-1 \leq X \leq 3) \geq 3/4$. Find the actual probability also.

Or

(b) (i) Let X be a random variable with p.d.f. $f(x) = \begin{cases} 1/3\; e^{-\frac{x}{3}}, & x > 0 \\ 0, & \text{otherwise.} \end{cases}$

Find (1) $P(X > 3)$ (2) M.G.F. of X (3) $E(X)$ (4) Var (X).

(ii) If a Poisson variate X is such that $P(X = 1) = 2P\,(X = 2)$, find $P(X = 0)$ and $Var(X)$. If X is a uniform random variable in [–2, 2], find the p.d.f. of $Y = |\,X\,|$ and $E\,(Y)$.

(a) (i) The joint p.d.f. of a bivariate R.V. (X, Y) is given by :

$$f(x, y) = \begin{cases} k\,xy, & 0 < x < 1,\ 0 < y < 1 \\ 0, & \text{otherwise,} \end{cases}$$

Where k is a constant (1) Find the value of K (2) Find $P(X + Y < 1)$ (3) Are X and Y independent random variables. Explain.

(ii) Let X and Y be independent standard normal random variables. Find the p.d.f. of $Z = \dfrac{X}{Y}$.

Or

(b) (i) If the Joint p.d.f. of random variables X and Y is $f(x, y) = \begin{cases} x\, e^{-x(1+y)}, & x > 0\ \ y > 0 \\ 0, & \text{otherwise,} \end{cases}$ find $f(y/x)$ and $E(Y/X = x)$.

(ii) Let X and Y be independent uniform random variables over (0, 1). Find the p.d.f. of $Z = X + Y$.

14. (a) (i) The one–step T.P.M. of a Markov chain $\{X_n\ ; n = 0, 1, 2 \ldots\}$ having state space $S = \{1, 2, 3\}$ is

$$P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$$ and the initial distribution is

$\pi_0 = (0.7, 0.2, 0.1)$. Find :

(1) $P(X_2 = 3 / X_0 = 1)$

(2) $P(X_2 = 3)$

(3) $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 1)$.

(ii) Discuss the reliability analysis of 2–unit system.

Or

(b) (i) Let $\{X_n; n = 1, 2, 3, \ldots\}$ be a Markov Chain with state space $S = \{0, 1, 2\}$ and one–step Transition Probability matrix.

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 1 & 0 \end{bmatrix}$$

(1) Is the chain ergodic? Explain.

(2) Find the invariant probabilities.

(ii) The life length of a device is exponentially distributed. It is found that the reliability of the device for 100 hours period of operation is 0.90. How many hours of operation is necessary to get a reliability 0.95?

15. (a) (i) A Petrol pump station has 2 pumps. The service times follow the exponential distribution with mean of 4 minutes and cars arrive for service is a Poisson Process at the rate of 10 cars per hour. Find the probability that a customer has to wait for service. What is the probability that the pumps remain idle?

(ii) Obtain the steady state probabilities for M/M/1/N/FCFS queueing model.

Or

(b) (i) In a given M/M/1 queueing system, the average arrivals is 4 customers per minute : $\rho = 0.7$. What are (1) mean number of customers L_s in the system (2) mean number of customers L_q in the queue (3) probability that the server is idle (4) mean waiting time W_s in the system.

(ii) Automatic car wash facility operates with only one bay. Cars arrive according to a Poisson process, with mean of 4 cars per hour and may wait in the facility's parking lot if the bay is busy. If the service time for all cars is constant and equal to 10 min, determine L_s, L_q, W_s and W_q

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