A 377

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2005.

Fifth Semester

Computer Science and Engineering

MA 038 — NUMERICAL METHODS

(Common to Polymer Technology, Metallurgical)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Using Newton–Raphson method, establish the formula $x_{n+1} = \frac{1}{2}\left(X_n + \frac{N}{x_n}\right)$ to calculate the squares root of N.

2. Compare Gauss–Elimination and Gauss–Seidel iteration methods of solving system of simultaneous equations.

3. Show that the second divided difference $[x_0, x_1, x_2]$ is independent of the order of the arguments.

4. State Newton's backward interpolation formula. When is it sued?

5. Explain the geometrical significance of trapezoidal rule.

6. What is the general Newton–Cotes quadrature formula? How is the trapezoidal rule its special case?

7. Compare and contrast Taylor series method and Runge–Kutta method in solving a first order ordinary differential equation numerically.

8. How many prior values are required to predict the next value in Adam's method?

9. Give an example of a parabolic equation.

10. What is meant by ill–posed problem in partial differential equation?

PART B — (5 × 16 = 80 marks)

11. (i) Solve the equation $\frac{dy}{dx} = 1 - y$ with the initial condition $x = 0$, $y = 0$, using Euler's algorithm and tabulate the solutions at $x = .1, .2, .3$ and $.4$.

(ii) Obtain the values of y at $x = 0.1, 0.2$ using Runge–Kutta method of fourth order for the differential equation $y' = -y$ given $y(0) = 1$.

12. (a) (i) Find, by Newton's method, the root of the equation $e^x = 4x$, which is approximately equal to 2, correct to three places of decimals.

(ii) Find the negative root of $x^3 - 2x + 5 = 0$.

Or

(b) (i) Find the largest eigen value and the corresponding eigen vector of $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

(ii) Solve

(1) by Gauss–Elimination method

(2) by Gauss–Jordan method, the equations

$$2x + y + 4z = 12$$
$$8x - 3y + 2z = 20$$
$$4x + 11y - z = 33.$$

13. (a) (i) Using a polynomial of third degree, complete the record given below of the export of a certain commodity during five years.

Year :	1917	1918	1919	1920	1921
Export (in tons) :	443	384	—	397	467

(ii) Determine the Hermite polynomial of degree 5 for the following data :

x :	2.0	2.5	3.0
$y = \ln x$:	0.69315	0.91629	1.09861
$y' = \frac{1}{x}$:	0.5	0.4	0.3333

Or

(b) (i) Probability distribution function values of a normal distribution are given as follows :

x :	0.2	0.6	1.0	1.4	1.8
$p(x)$:	0.39104	0.33322	0.24197	0.14973	0.07895

Using a suitable interpolation formula, find the value of $p(1.2)$.

(ii) Given :

θ :	0°	5°	10°	15°	20°	25°	30°
$\tan\theta$:	0	0.0875	0.1763	0.2679	0.3640	0.4663	0.5774

Show that $\tan 16° = 0.2867$. Use Stirling's formula.

14. (a) (i) From the following table, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for $x = 1.2$.

x :	1.0	1.2	1.4	1.6	1.8	2.0	2.2
y :	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250

(ii) A solid of revolution is formed by rotating about the x–axis the area between the x–axis, the lines $x = 0$ and $x = 1$ and a curve through the points with the following coordinates.

x :	0.00	0.25	0.50	0.75	1.00
y :	1.0000	0.9896	0.9589	0.9089	0.8415

Estimate the volume of the solid formed, giving the answer to three decimal places.

Or

(b) (i) Evaluate $\int_0^1 \int_0^1 e^{x+y}\, dx\, dy$ using the trapezoidal and Simpson's rules.

(ii) Find $\int_0^1 x\,dx$ by Gaussian quadrature formula.

15. (a) Solve $u_{xx} + u_{yy} = 0$ over the square mesh of side 4 units satisfying the following boundary conditions :

(i) $u(0, y) = 0$ for $0 \le y \le 4$

(ii) $u(4, y) = 12 + y$ for $0 \le y \le 4$

(iii) $u(x, 0) = 3x$ for $0 \le x \le 4$

(iv) $u(x, 4) = x^2$ for $0 \le x \le 4$.

Or

(b) Solve the equation $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ subject to the following conditions :

$$\left.\begin{array}{l} u(0, t) = 0 \\ u(1, t) = 0 \end{array}\right\} (t > 0)$$

$$\left.\begin{array}{l} \frac{\partial u}{\partial t}(x, 0) = 0 \\ \text{and } u(x, 0) = \sin^3 \pi x \end{array}\right\} \text{for all } x \text{ in } 0 \le x \le 1$$