B 471

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2005.

Second Semester

Computer Science and Engineering

MA 035 — DISCRETE MATHEMATICS

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Define contrapositive of a statement.

2. Give the symbolic form of the statement "every book with a blue cover is a mathematics book".

3. State the principle of mathematical induction.

4. Find the coefficient of x^{10} in $(1+x^5+x^{10}+...)^3$.

5. Find all the cosets of the subgroup $H=\{1,-1\}$ in $G=\{1,-1,i,-i\}$ with the operation multiplication.

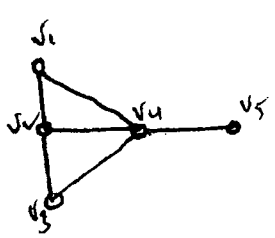
6. Give an example of a field with three elements.

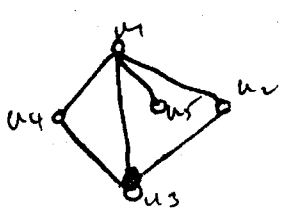
7. Define partial order relation.

8. Give an example of non distributive lattice.

9. Define pendentant vertex in a graph.

10. Whether the following graphs are isomorphic or not.





PART B — ($5 \times 16 = 80$ marks)

11. (i) Give the principal disjunctive and conjunctive normal forms of the expression

$$(\neg P \to R) \cap (q \leftrightarrow p)$$

(ii) Show that $R \to S$ can be derived from the premises $p \to (q \to s)$, $\neg R \cup P$ and q

12. (a) (i) Prove by induction for $n \geq 1$ (6)

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

(ii) Solve the recurrence relation (10)

$$F(n) - F(n-1) - F(n-2) = 0,\ F(0) = 1,\ F(1) = 1.$$

Or

(b) (i) In a survey of 100 students, it was found that 40 studied Mathematics, 64 studied Physics 35 studied Chemistry, 1 studied all the three subjects 25 studied Mathematics and Physics, 3 studied Mathematics and Chemistry and 20 studied Physics and Chemistry. Find the number of students who studied Chemistry only. (6)

(ii) Using the generating function method solve the difference equation $y_{n+2} - y_{n+1} - 6y_n = 0$ given that $y_1 = 1,\ y_0 = 2$. (10)

13. (a) (i) If every element of a group is its own inverse prove that G is abelian. Is the converse is true? (6)

(ii) Let $(G, *)$ and (H, Δ) be two groups. Let $g : G \to H$ be a homomorphism. Let H' denote the image set of G under the mapping g and k be Ker (g). Prove that G/K is isomorphic to H'.(10)

Or

(b) (i) State and prove Lagranges theorem. (12)

(ii) Define ring and give an example of a ring with zero-divisors. (4)

14. (a) (i) Draw the Hasse diagram for the partial ordering $\{(A, B)/A \subseteq B\}$ on the power st $P(s)$ where $S = \{a, b, c\}$.

(ii) In any lattice prove that

$$a \wedge (b \cup c) \geq (a \wedge b) \cup (a \wedge c)$$

Or

(b) (i) Prove that in a distributive lattice every element has unique complement.

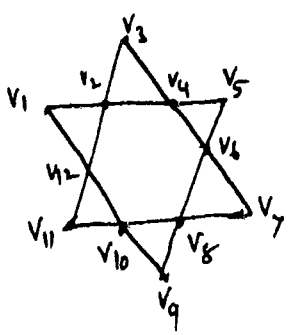
(ii) Simplify the following Boolean function

$$f(x_1, x_2, x_3, x_4) = x_1 x_3 + x'_1 x_3 x_4 + x_2 x'_3 x_4 + x'_2 x_3 x_4$$

by any method.

15. (a) (i) Prove that the total degree of vertices in a graph is equal to twice the edges of the graph.

(ii) Which of the following are Euler graphs :



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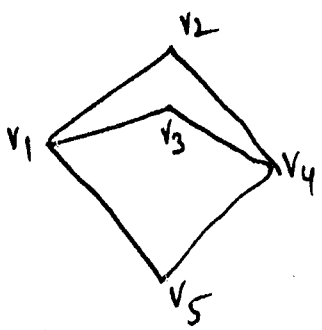

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Or

(b) (i) Give an example of a graph which is Hamiltonian but not Eulerian and vice-versa.

(ii) Draw the directed graph G corresponding to the adjacency matrix

$$A = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}.$$

Also find the indegree and outdegree for each vertices and verify that total indegree equal to total outdegree = the number of edges.
