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**J 5245**

M.E. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2007.

First Semester

Communication Systems

MA 151 — APPLIED MATHEMATICS FOR ELECTRONICS ENGINEERS

(Common to M.E. — Applied Electronics, M.E. Optical Communication, M.E. VLSI Design and M.E. Digital Communication and Network Engineering)

(Regulation 2002)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the characteristics of  $U_{xx} + 2U_{xy} + U_{yy} = 0$ .
2. Write all variables separable solutions of wave equation.
3. Write Bessel's differential equation of order  $n$  and define Bessel functions of order  $n$ .
4. Express  $x^3 + 2x^2 - x - 3$  in terms of Legendre polynomials.
5. Prove that  $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$ .
6. If  $A, B$  and  $C$  are any 3 events such that  $P(A) = P(B) = P(C) = \frac{1}{4}$ ,  $P(A \cap B) = P(B \cap C) = 0$ ,  $P(C \cap A) = \frac{1}{8}$ . Find the probability that at least one of the events  $A, B$  and  $C$  occurs.
7. The joint pdf of the  $R \vee (X, Y)$  is  $f(x, y) = 4xy e^{-(x^2+y^2)}$ ,  $x > 0$ ,  $y > 0$ . Prove that  $X$  and  $Y$  are independent.
8. If  $X$  is uniformly distributed in  $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ , find the pdf of  $Y = \tan X$ .
9. State Little's formulas.
10. Find an expression to compute average queue length in  $(M/M/1):(\infty/\text{FIFO})$ .

11. (a) Solve the initial boundary value problem given, using Laplace Transform method :

$$u_{tt} = u_{xx}, 0 < x < 1, t > 0$$

$$u(0, t) = u(1, t) = 0, t > 0$$

$$u(x, 0) = \sin \pi x, u_t(x, 0) = -\sin \pi x, 0 < x < 1. \quad (16)$$

Or

- (b) Solve  $\phi_{tt} = c^2 \phi_{xx}$  in  $0 < x < a, t > 0$  subject to  $\phi(x, 0) = \begin{cases} \frac{x}{b}, & 0 \leq x \leq b \\ \frac{a-x}{a-b}, & b < x \leq a \end{cases}$

$$\frac{\partial \phi}{\partial t}(x, 0) = 0, 0 < x < a$$

$$\phi(0, t) = \phi(a, t) = 0, t \geq 0 \quad (16)$$

12. (a) (i) Express  $J_5(x)$  in terms of  $J_0(x)$  and  $J_1(x)$ . (8)

- (ii) Prove that  $(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$ . (8)

Or

- (b) (i) Solve the differential equation

$$y'' + \frac{y'}{x} + 8 \left( -\frac{1}{x^2} \right) y = 0 \quad (6)$$

- (ii) If  $f(x) = \begin{cases} 0, & -1 < x \leq 0 \\ x, & 0 < x < 1, \end{cases}$  (10)

$$\text{Show that } f(x) = \frac{1}{4} P_0(x) + \frac{1}{2} P_1(x) + \frac{5}{16} P_2(x) - \frac{3}{32} P_4(x) + \dots$$

13. (a) (i) If  $A, B$  and  $C$  are random subsets (events) in a sample space and if they are pairwise independent and  $A$  is independent of  $(B \cup C)$ , prove that  $A, B$  and  $C$  are mutually independent. (6)

- (ii) The joint pdf of a two-dimensional  $R \vee (X, Y)$  is given by

$$f(x, y) = xy^2 + \frac{x^2}{8}, 0 \leq x \leq 2, 0 \leq y \leq 1. \text{ Compute}$$

(1)  $P(X > 1)$

(2)  $P\left(Y < \frac{1}{2}\right)$

$$(3) \quad P\left(X > 1/Y < \frac{1}{2}\right)$$

$$(4) \quad P(X + Y \leq 1)$$

$$(5) \quad P(X < Y). \quad (10)$$

Or

- (b) (i) If the joint pdf of  $(X, Y)$  is given by  $f_{XY}(x, y) = x + y, 0 \leq x, y \leq 1$ , find the pdf of  $U = XY$ . (10)

- (ii) Prove that the angle between the lines of regression  $\cdot \theta$  is given by 
$$\tan \theta = \left(\frac{1-r^2}{r}\right) \left(\frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}\right). \quad (6)$$

14. (a) (i) Find the coefficient of correlation between  $x$  and  $y$  using the given data. What will be the expected value of  $y$  when  $x = 35$ ? (8)

$x$ : 27 28 29 30 32 32 33

$y$ : 17 18 19 19 21 20 21

- (ii) The point pdf of  $(X, Y)$  is given by 
$$f(x, y) = \frac{9(1+x+y)}{2(1+x)^4(1+y)^4}, x > 0, y > 0$$
 find the marginal densities of  $X$  and  $Y$ . Are they independent? (8)

Or

- (b) (i) Find the moment generating function of the binomial distributions and hence find its mean and variance. (8)

- (ii) State and prove Rodrigue's formula. (8)

15. (a) Customers arrive at a one-man barber shop according to a Poisson process with a mean inter arrival time of 12 minutes customers spend an average of 10 minutes in the barber's chair.

- (i) What is the expected number of customers in the barber shop and in the queue?
- (ii) Calculate the average of time an arrival can walk straight into the barber's chair without having to wait.
- (iii) How much time can a customer expect to spend in the barber's shop?
- (iv) What is the average time customer spend in the queue?
- (v) What is the probability that more than 3 customers are in the system? (16)

Or

- (b) A petrol pump station has 4 pumps. The service times follow the exponential distribution with a mean of 6 minutes and cars arrive for service in a Poisson process at the rate of 30 cars per hour.
- (i) What is the probability that an arrival would have to wait in line?
  - (ii) Find the average waiting time, average time spent in the system and the average number of cars in the system.
  - (iii) For what percentage of time would a pump be idle on an average?
- (16)
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