Reg. No. : | | | | | | | | | | | |

W 6291

M.E. DEGREE EXAMINATION, JANUARY 2008.

First Semester

Structural Engineering

MA 1602 — APPLIED MATHEMATICS

(Common to M.E. – Soil Mechanics and Foundation Engineering)

(Regulation 2005)

Time : Three hours   Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Define Laplace transform.

2. Compute $L\{e^{ax}\sin bx\}$.

3. Discuss Ritz method of calculus of variations.

4. Find the Euler equation associated with $\int_{x_0}^{x_1}(16y^2-(y'')^2+\varphi(x))dx$, where $\varphi(x)$ is an arbitrary continuous function of x.

5. If $f(x, y)=e^{-(x+y)}$, $0<x<\infty, 0<y<\infty$

   Find $P(1<X+Y<2)$

6. The random variables X and Y are jointly normally distributed and U and V are defined by $U=X\cos\alpha+Y\sin\alpha, V=Y\cos\alpha-X\sin\alpha$. Show that U and V will be uncorrelated is $\tan 2\alpha=\dfrac{2r\sigma_X\sigma_Y}{\sigma_X^2-\sigma_Y^2}$.

7. Define partial and multiple correlations.

8. Explain the method of moments for estimating the parameters.

9. State a more accurate formula for calculating the number of unique schematic contained in a randomly generated population of size m when the string length is L.

10. Define neural network. Mention its uses.

PART B — (5 × 16 = 80 marks)

11. (a) Find $L\{\sin \lambda t\}$. Using the result, show that

(i) $$\frac{s}{\left(s^2+\lambda^2\right)^2} = L\left\{\frac{t}{2\lambda}\sin \lambda t\right\}$$ (8)

(ii) $$\frac{1}{(s^2+\lambda^2)^2} = L\left\{\frac{1}{2\lambda^3}(\sin \lambda t - \lambda t \cos \lambda t)\right\}.$$ (8)

Or

(b) Solve by Laplace transform the boundary value problem, $u_{xx} = u_{tt}$, $0 < x < \infty$, $t > 0$, $u_x(0, t) = 0$ $u(x, t) \to 0$ as $x \to \infty$. $u(x,0) = e^{-x}$ and $u_t(x, 0) = 0$.

12. (a) Find the extremals of the functional $J[y(x), z(x)] = \int_0^{\pi} (2yz - 2y^2 + y'^2 - z'^2)\, dx$ if $y(0) = 0$, $y(\pi) = 1$, $z(0) = 0$ and $z(\pi) = -1$.

Or

(b) Use Ritz method to the problem of minimizing the integral

$I = \int_0^1 [(y')^2 - y^2 - 2xy]dx$, $y' = \frac{dy}{dx}$ subject to the boundary conditions $y(0) = y(1) = 0$.

13. (a) (i) A variable X is distributed at random between the values 0 and 4 and its probability density function is given by $f(x) = kx^3(4-x)^2$. Find the value of k, the mean and standard deviation of the distribution.

(ii) A random variable X has the probability law

$$dF(x) = \frac{x}{b^2} e^{-x^2/2b^2}\, dx,\ 0 \le x < \infty$$

Find the distance between the quartiles and show that the ratio of this distance to the standard deviation of X is independent of the parameter 'b'.

Or.

(b) If X and Y are standard normal variate with co-efficient of correlation ρ, show that

(i) Regression of Y on X is linear

(ii) $X + Y$ and $X - Y$ are independently distributed

(iii) $\varphi = \dfrac{X^2 - 2\rho + XY + Y^2}{(1-\rho^2)}$ is distributed as a chi-square.

14. (a) (i) Explain briefly the method of maximum likelihood estimation and also mention its properties.

(ii) Find the M.L.E of the parameters α and λ for the following distribution

$$f(x; \alpha, \lambda) = \frac{1}{\Gamma(\lambda)} \left(\frac{\lambda}{\alpha}\right)^{\lambda} e^{-\lambda x/\alpha} x^{\lambda - 1},\ 0 \le x < \infty,\ \lambda > 0\,.$$

Or

(b) If all the correlation co-efficient of new order in a set of p-variates are equal to ρ, show that

(i) every partial correlation of s^{th} order is $\dfrac{\rho}{1 + s\rho}$

(ii) the co-efficient of multiple correlation R of a variate with the other $(p-1)$ variates is given by $1 - R^2 = (1 - \rho)\left[\dfrac{1 + (p-1)\rho}{1 + (p-2)\rho}\right]$.

15. (a) (i) Explain briefly fuzzy relations and operations on fuzzy relations. Mention the properties of fuzzy relations.

(ii) A spray-on foam deposition process can be modeled by the following relationship $T \propto (F)(P)$, whose T = thickness of deposited foam; F = flow rate is mass per unit time; P = period of time under spray gun. In general, each independent factor can have a tolerance (fuzziness) of ± 20 per cent, each factors normalize fuzzy set is

$$F = \left\{\frac{0.9}{0.8} + \frac{1}{1} + \frac{0.9}{1.2}\right\}$$

$$P = \left\{\frac{0.9}{0.8} + \frac{1}{1} + \frac{0.9}{1.2}\right\}$$

Find the fuzzy relation $T = F \times P$.

Or

(b) Develop a statistical routine to track population fitness standard deviation and then create a routine for sigma (σ) truncation.