

PART B — (5 × 16 = 80 marks)

11. (a) For the following bivariate probability distribution of X and Y

		Y	1	2	3	4	5	6	
X			0	0	0	$\frac{1}{32}$	$\frac{2}{32}$	$\frac{2}{32}$	$\frac{3}{32}$
	1	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	
	2	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	0	$\frac{2}{64}$	

- find (i) $P(X \leq 1, Y = 2)$
 (ii) $P(X \leq 1)$
 (iii) $P(Y = 3)$
 (iv) $P(Y \leq 3)$
 (v) $P(X < 3, y \leq 4)$
 (vi) $P(X \leq 1 / y \leq 3)$
 (vii) $P(X + Y \leq 4)$. (16)

Or

- (b) The joint density function of continuous random variables X and Y is

$$f(x, y) = \begin{cases} k(x + y), & 0 < x < 1, 0 < y < 2 \\ 0, & \text{elsewhere} \end{cases}$$

Find the value of k and hence ρ_{xy} . ρ_{xy} denotes the correlation coefficient of x and y . (16)

12. (a) Show that $1 - R_{1.23}^2 = (1 - r_{12}^2)(1 - r_{13.2}^2)$

Deduce that

- (i) $R_{1.23} \geq r_{12}$
 (ii) $R_{1.23}^2 = r_{12}^2 + r_{13}^2$, if $r_{23} = 0$
 (iii) $1 - R_{1.23}^2 = \frac{(1 - \rho)(1 + 2\rho)}{(1 + \rho)}$.

Provided all the coefficients of zero order are equal to ρ .

- (iv) If $R_{1.23} = 0$, X_1 is uncorrelated with any of the other variables, ie $r_{12} = r_{13} = 0$. (16)

Or

- (b) (i) Prove that the maximum likelihood estimate of the parameter α of a population having the density function $\frac{2}{\alpha^2}(\alpha - x)$, $0 < x < \alpha$ for a sample of unit size $s = 2x$, x being the sample value. Show also that the estimate is biased. (8)
- (ii) Find the maximum likelihood estimate for the parameter λ of a Poisson distribution on the basis of a sample of size n . Also find its variance. (5 + 3)

13. (a) A car rental agency, which uses 5 different brands of tyres in the process of deciding the brand of tyre to purchase as standard equipment for its fleet, finds that each of 5 tyres of each brand lasts the following number of kilometers (in thousands)

Tyre brands				
A	B	C	D	E
36	46	35	45	41
37	39	42	36	39
42	35	37	39	37
38	37	43	35	35
47	43	38	32	38

Test the hypothesis that the five tyre brands have almost the same average life. (16)

Or

- (b) The following table gives for a sample of married women, the level of education and the marriage adjustment score.

Level of education	Marriage adjustment				Total
	Very low	Low	High	Very High	
College	24	97	62	58	241
High school	22	28	30	41	121
Middle school	32	10	11	20	73
Total	78	135	103	119	435

Can you conclude from the above data that higher the level of education, the greater is the degree of adjustment in marriage? Use 5% Level of significance. (16)

14. (a) An experiment was designed to study the performance of four different detergents for cleaning fuel injectors. The following "Cleanness" readings were obtained with specially designed equipment for 12 tanks of gas distributed over three different models of engines :

	Engine 1	Engine 2	Engine 3
Detergent A	45	43	51
Detergent B	47	46	52
Detergent C	48	50	55
Detergent D	42	37	49

Looking on the detergents as treatments and the engines as blocks, obtain the appropriate ANOVA table to test at the 0.01 level of significance whether there are differences in the detergents or in the engines. (16)

Or

- (b) Analyse the variance in the following Latin square of yields (in kgs) of paddy where A, B, C, D denote the different methods of cultivation.

D 122	A 121	C 123	B 122
B 124	C 123	A 122	D 125
A 120	B 119	D 120	C 121
C 122	D 123	B 121	A 122

Examine whether the different methods of cultivation have given significantly different yields. (16)

15. (a) Calculate the seasonal indices by the ratio and moving average method from the following data : (16)

Wheat prices in Rupees per Quintal				
Year	1994	1995	1996	1997
Quarters				
Q ₁	75	86	90	100
Q ₂	60	65	72	78
Q ₃	54	63	66	72
Q ₄	59	80	85	93

Or

- (b) The sales of a company for the last eight years are given below :
- | | | | | | | | | |
|--------------------|------|------|------|------|------|------|------|------|
| Year : | 1980 | 1981 | 1982 | 1983 | 1984 | 1985 | 1986 | 1987 |
| Sales (Rs('000)) : | 52 | 45 | 98 | 92 | 110 | 185 | 175 | 220 |

Estimate sales figure for 1988 using an equation of the form $Y = ab^X$ where X = years and Y = sales. (16)