E 6050

M.E. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2005.

First Semester

Control and Instrumentation

CI 131 — SYSTEM THEORY

(Common to M.E. – Power Electronics and Drives and M.E. – Power Systems Engineering)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Show that the estimation error dynamics of the following system cannot be made faster than 1 sec.

$$X = \begin{bmatrix} 0 & 1 \\ -10 & -11 \end{bmatrix} X + \begin{bmatrix} 0 \\ 1 \end{bmatrix} U$$

$$Y = [1 \quad 1]x.$$

2. Show that pole-zero cancellation occurs in the transfer function model of an unobservable system.

3. Obtain the unit step output response of the system with zero initial conditions and show that only one of the mode is excited in the output.

$$X = \begin{bmatrix} 0 & 1 \\ -30 & -11 \end{bmatrix} X + \begin{bmatrix} 0 \\ 1 \end{bmatrix} U$$

$$Y = [1 \quad 6]x$$

4. Obtain the minimal realization of the system defined by the state equations

$$X = \begin{bmatrix} -4 & -4 \\ -4 & -4 \end{bmatrix} X + \begin{bmatrix} 1 \\ 1 \end{bmatrix} U$$

$$Y = [1 \quad 1]X$$

5. Consider the system given by $G(s) = \dfrac{(s^7 + s^3 + 2s + 5)}{s^8 + s^5 + 2s^3 + 10}$. Obtain a state model such that all the states are completely controllable.

6. Compare the relative merits and demerits of state and output feedback.

7. Consider the non-linear equation $\frac{dx}{dt} = x^2 + 5xu + 2u^2$. Obtain a linearised state model around the operating point say x_0 and u_0.

8. Given $G(s) = (s+z)/[(s+3)(s+0.5)]$. Obtain a first order equivalent for the two cases namely, when $z > 0.5$ and $z < 3$. Also comment on the stability of the two resultant reduced order models.

9. The describing function analysis makes an assumption that the higher order harmonics produced by the non-linear part of the system is filtered by the linear part. Justify this assumption.

10. What type of frequency responses are used in analysis of MIMO systems? Why?

PART B — (5 × 16 = 80 marks)

11. Consider the system described by the state equation.

$$x = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 5 & 0 \\ 0 & 2 & 3 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} u$$

(i) Derive the transformation that transforms the system to controllable canonical form. (6)

(ii) Design a state-feedback controller to assign the closed loop poles at $-1, -5$ and -3. (6)

(iii) State whether there is any freedom in choosing the state feedback gain matrices. (4)

12. (a) A MIMO system is described by the Transfer function matrix.

$$G(s) = \begin{bmatrix} \frac{s+1}{(s+2)(s+1)} & \frac{1}{(s+2)(s+1)} \\ \frac{s}{(s+2)(s+1)} & \frac{2}{(s+1)(s+5)} \end{bmatrix}$$

(i) Obtain a minimal state model. (6)

(ii) Check the controllability and observability of the system using Kalman's test. (6)

(iii) Distinguish the terms Stabilisability and controllability. (4)

Or

(b) Consider the system whose state equation is given by

$$X = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix} X + \begin{bmatrix} 2 \\ 2 \end{bmatrix} U$$

$$y = \begin{bmatrix} 2 & 1 \end{bmatrix} X$$

(i) Show through Kalman's test and controllability Grammian test that it is possible to transform the state from any a given initial state to arbitrary final value in finite time. (8)

(ii) Derive the control law that will transform the state from $x(0) = [1 \ \ 2]$ to $x(2) = [-1 \ \ 1]$. (8)

13. (a) (i) Distinguish the terms, decoupling zeros, invariant zeros and transmission zeros of a MIMO system. (6)

(ii) Determine the zeros of the system whose state equations are given by (10)

$$X = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ -6 & -11 & -6 \end{bmatrix} X + \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix} u \ .$$

$$Y = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1 & 1 \end{bmatrix} X$$

Or

(b) Consider the transfer function matrix of a linear time invariant system

$$G(s) = \begin{bmatrix} 1/(s+2) & 1/[(s+1)(s+3)] \\ (s+5)/(s+2) & (5s+5)/(s+3) \end{bmatrix}$$

(i) Obtain a minimal realisation of the system and transform it to observable canonical form. (8)

(ii) Design a suitable observer so that the states are available for estimation in one second. (8)

14. (a) Consider the Vander Pal's equation $\ddot{X} + 2\mu(1 - X^2)X + X = 0$.

(i) Draw the phase trajectories and

(ii) Show that stable and unstable limit cycles occur in the phase plane depending on the sign of μ.

Or

(b) (i) Determine the stability of the equilibrium point of the system whose state equation is given by $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -x_1 + x_2 \\ x_1 - x_2 - x_2^3 \end{bmatrix}$. (10)

(ii) Explain the Liapunov equation method of assessing stability when the non-linear part of the system described in 14 (b) (i) is zero. (6)

15. (a) Obtain the phase trajectory starting from (–1 –2) with e and de/dt as state variables for the system shown in figure 15 (a) for the three cases namely (i) $kp = 0$ and $kd = 10$ (ii) $kp = 10$ and $kd = 0$ and (iii) $kp = 1$ and $kd = 1$. (16)

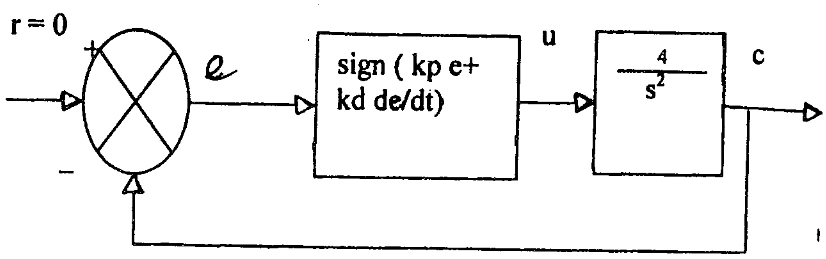
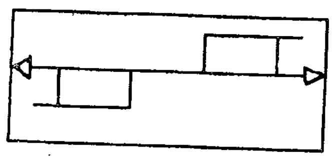


Fig. 15 (a)

Or

(b) (i) Derive the describing function for the relay whose input/output characteristics is given by figure 15 (b) (i) Let d be the width of dead zone and h be the width of hysterisis. (8)



Relay with hysterisis and dead zone

Fig. 15 (b) (i)

(ii) Distinguish the time domain and frequency domain model reduction techniques. (8)