G 7109

M.E. DEGREE EXAMINATION, JANUARY 2006.

First Semester

Power Systems Engineering

MA 1614 — APPLIED MATHEMATICS FOR ELECTRICAL ENGINEERS

(Common to M.E. Control and Instrumentation, M.E. Power Electronics and Drives and M.E. High Voltage Engineering.)

(Regulation 2005)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the generalised eigen vector of rank 2 corresponding to $\lambda = 5$ for the matrix $A = \begin{pmatrix} 5 & 1 & 0 \\ 0 & 5 & 1 \\ 0 & 0 & 5 \end{pmatrix}$

2. Define pseudo inverse of a matrix A.

3. Write down the differential equation with constant coefficient corresponding to the functional $\int_{x_0}^{x_1} \left(x^2 y'^2 + 2y^2 + 2xy \right) dx$.

4. Test for an extremum of the functional $\int_{x_0}^{x_1} \left(y^2 + 2x\, y\, y' \right) dx$.

5. Define basic solution and non – degenerate basic solution in L.P.P.

6. Write down the dual of the following primal problem min $z = 15x_1 + 12x_2$ subject to the constraints $x_1 + 2x_2 \geq 3$, $2x_1 - 4x_2 \leq 5$, $x_1,\, x_2 \geq 0$.

7. State Bellman's principle of optimality.

8. Write down the characteristics of dynamic programming.

9. Prove that $P[X(t) = r] = \dfrac{e^{-\lambda t}(\lambda t)^r}{r!}$ $r = 0, 1, 2, \cdots$ is not covariance stationary.

10. Prove that the spectral density function of a real random process is an even function.

11. (i) Construct QR decompositions for the matrix

$$A = \begin{pmatrix} -4 & 4 & 2 \\ 4 & -4 & 1 \\ 2 & 1 & 0 \end{pmatrix}$$ (8)

(ii) Using least square approximation, solve the following system of equation. (8)

$$x_3 + 2x_4 = 1$$
$$x_1 + 2x_2 + 2x_3 + 3x_4 = 2$$

12. (a) (i) Find the path on which a particle in the absence of friction will slide from one point to another in the shortest time under the action of gravity. (8)

(ii) Find the geodesics on a right circular cylinder of radius a. (8)

Or

(b) (i) Find the extremals of the isoperimetric problem $\int_0^1 \left(y'^2 + x^2 \right) dx$ given that $\int_0^1 y^2 dx = 2$ and $y(0) = 0, y(1) = 0$. (8)

(ii) Solve the boundary value problem

$y'' + y + x = 0$, $y(0) = 0$, $y(1) = 0$ by Rayleish – Ritz method. (8)

13. (a) (i) Use simplex method to solve the following L.P.P.

Max $z = x_1 - 3x_2 + 2x_3$ subject to the conditions

$3x_1 - x_2 + 2x_3 \leq 7$, $-2x_1 + 4x_2 \leq 12$,

$-4x_1 + 3x_2 + 8x_3 \leq 10$, $x_1, x_2, x_3 \geq 0$. (8)

(ii) Find the optimum integer solution to the following L.P.P

Max $z = x_1 + 2x_2$ subject to

$2x_2 \leq 7$, $x_1 + x_2 \leq 7$, $2x_1 \leq 11$, $x_1, x_2 \geq 0$ and are integers. (8)

Or

(b) (i) Use dual simplex method to solve the L.P.P.

Max $z = 3x_1 + x_2$ subject to $x_1 + x_2 \geq 1$,

$2x_1 + 3x_2 \geq 2$, $x_1, x_2 \geq 0$. (8)

(ii) Using two phase method, solve the L.P.P

Max $z = 5x - 2y + 3z$ subject to

$2x + 2y - z \geq 2$, $3x - 4y \leq 3$, $y + 3z \leq 5$, $x, y, z \geq 0$. (8)

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14. (a) (i) Obtain the set of necessary conditions for the non–linear programming

$$\text{Max } z = x_1^2 + 3x_2^2 + 5x_3^2$$

Subject to $x_1 + x_2 + x_3 = 2$

$$5x_1 + 2x_2 + x_3 = 5 \quad x_1, x_2, x_3 \geq 0.$$ (4)

(ii) By dynamic programming technique, solve the problem

$$\text{Min } z = x_1^2 + x_2^2 + x_3^2 \text{ subject to the}$$

Constraints $x_1 + x_2 + x_3 \geq 15$, and $x_1, x_2, x_3 \geq 0$. (12)

Or

(b) Using Kuhn – Tucker conditions solve the following L.P.P.

$$\text{Max } z = -x_1^2 - x_2^2 - x_3^2 + 4x_1 + 6x_2 \quad \text{subject to the constraints}$$
$x_1 + x_2 \leq 2$, $2x_1 + 3x_2 \leq 12$ and $x_1, x_2 \geq 0$. (16)

15. (a) (i) A stationary zero mean random process $x(f)$ has the auto correlation function $R_{xx}(z) = 10\, e^{-0.1z^2}$, find the mean and variance of $\bar{X}_T = \dfrac{1}{5} \int_0^5 x(f)dt$. (8)

(ii) If $X(t) = A \cos wt + B \sin wt$ where A and B are random process with $E(A) = E(B) = 0$, $E(A^2) = E(B^2)\, E(AB) = 0$, then prove that $X(t)$ is a ergodic process. (8)

Or

(b) (i) Three boys A, B and C are throwing a ball to each other A always throw the ball to B and B throws to C but C is just as likely to throw the ball to B as to A show that the process is Markosian and hence find the transition matrix and classify the states. (8)

(ii) Find the power spectral density of a WSS process with auto correlation function $R(z) = e^{-\alpha z^2}$. (8)

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