G 7112

M.E. DEGREE EXAMINATION, JANUARY 2006.

First Semester

Computer Science and Engineering

MA 1617 — MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE

(Regulation 2005)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Define a partially ordered set.

2. Give an example for a bijective mapping.

3. Find the truth table of $\sim (p_{\vee} \sim Q)$.

4. Negate the statement $(\forall x)p(x) \wedge (\exists y)p(y)$.

5. What is a pigeonhole principle?

6. Obtain the closed form expression of generating function for the sequence $(3^0, 3^1, 3^2, 3^3, \ldots)$.

7. Define ambiguous grammar.

8. Define finite automation.

9. What is the probability of obtaining 53 Sundays in a leap year?

10. What is the expectation of getting the numbers in rolling a die?

PART B — (5 × 16 = 80 marks)

11. (i) Obtain the mean and variance of binomial probability distribution. (8)

(ii) In a bolt factory machines A, B, C manufacture respectively 25, 35 and 40 percent of the total of their output 5, 4 and 2 percent are defective bolts. A bolt is drawn at random from the product and is found to be defective. What are the probabilities that it was manufactured by machines A, B or C? (8)

12. (a) (i) Let A, B and C be sets. Prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$. (8)

(ii) Let $f : A \to B$, $g : B \to C$ be bijection. Prove that $g \circ f : A \to C$ is also a bijection. (8)

Or

(b) (i) Let $S = \{1, 2, 3, 4, 5\}$ and let $A = S \times S$. Define the following relation R on $A : (a, b)\, R (a', b')$ if and only if $ab' = a'b$.

(1) Show that R is an equivalence relation.

(2) Compute A / R. (8)

(ii) (1) Show that $A \cup (A \cap B) = A = A \cap (A \cup B)$.

(2) Give an example of sets A and B such that $A \times B \neq B \times A$. Justify. (4 + 4)

13. (a) (i) Show that $\neg P$ is a valid conclusion from the premises $R \to \neg Q$, $R \vee S$, $S \to \neg Q$ and $P \to Q$. (8)

(ii) Using indirect method, prove that $(P \wedge Q) \wedge R \Leftrightarrow P \wedge (Q \wedge R)$. (8)

Or

(b) (i) Construct the truth table for $Q \wedge (P \to Q) \to P$. (4)

(ii) Obtain a disjunctive normal form of $\neg (P \vee Q) \leftrightarrow (P \wedge Q)$. (4)

(iii) Obtain the principal disjunctive normal form for $(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$. (8)

(a) Solve $S(K) - 4S(K-1) + 4S(K-2) = 3K + 2^K$. $S(0) = 1$, $S(1) = 1$. (16)

Or

(b) (i) Show that if $f(x,y)$ defines the remainder upon division of y by x, then it is a primitive recursive function. (8)

(ii) Obtain the number of permutations of all the letters of the words.

(1) Committee.

(2) Engineering. (3 + 3)

(iii) What is the principle of counting? (2)

5. (a) Construct a grammar generating $L = \{ a^n b^n c^n : n \geq 1 \}$. (16)

Or

(b) Explain in detail about Turing machines. (16)
