E 8238

M.C.A. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2005.

First Semester

MA 154 — MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE

Time : Three hours   Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Negate the statement : 4 is a prime number or 3^2 is an even integer.

2. Show the following using the automatic theorem

$$P \Rightarrow (\neg P \to Q).$$

3. In how many ways can ten adults and five children stand in a line so that no two children are next to each other?

4. State the general principle of Inclusion and Exclusion.

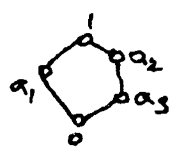
5. Prove that monoid homomorphism preserves invertibility.

6. Define a cyclic group.

7. Show that $f(x,y) = x^y$ is a primitive recursive function.

8. Define Turing computable function with example.

9. Is the Lattice given by the diagram distributive? Justify.



10. In a Boolean algebra, prove the complement of any element is unique.

PART B — (5 × 16 = 80 marks)

11. (i) Show that the proper subtraction $m \dot{-} n$ is computable by designing a Turing machine that computes $m \dot{-} n$. (12)

(ii) Is the function $f(x) = \frac{x}{2}$ a recursive function? (4)

12. (a) (i) Prove the following without using truth table :

$$(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R.$$ (6)

(ii) Show that

$$(x)(P(x) \vee Q(x)) \Rightarrow (x)\, P(x) \vee (\exists x)\, Q(x).$$ (10)

Or

(b) (i) Obtain the principal disjunctive normal form and principal conjunctive normal form of the formula.

$$P \vee (\neg P \rightarrow (Q \vee (\neg Q \rightarrow R)))$$ (8)

(ii) Show that the following set of premises is inconsistent. "If the contract is valid, then John is liable for penalty. If John is liable for penalty, he will go bankrupt. If the bank will loan him money, he will not go bankrupt. As a matter of fact, the contract is valid and the bank will loan him money". (8)

13. (a) (i) State pigeon-hole principle (strong form). Using it prove that in any group of six people, at least three must be mutual friends or at least three must be mutual strangers. (6)

(ii) Solve the recurrence relation using generating function

$y_{n+2} - 4y_{n+1} + 3y_n = 0$, with $y_0 = 2, y_1 = 4$. (10)

Or

(b) (i) Show that $n^2 > 2n + 1$ for $n \geq 3$ by mathematical induction. (6)

(ii) How many solutions does $x_1 + x_2 + x_3 = 11$ have, where x_1, x_2 and x_3 are non-negative integers with $x_1 \leq 3, x_2 \leq 4$ and $x_3 \leq 6$? (Use principle of Inclusion and Exclusion). (10)

14. (a) (i) Let $f : S \to T$ be a homomorphism of the semi group $(S,*)$ onto the semi group $(T,*')$. Let R be the relation on S defined by aRb if and only if $f(a) = f(b)$, for a and b in S. Prove the following :

(1) R is a congruence relation.

(2) $(T, *')$ and the quotient semi group $(S/R, \circledast)$ are isomorphic. (10)

(ii) Prove that intersection of any two normal subgroups of a graph is a normal subgroup. (6)

Or

(b) (i) Define the symmetric group S_n of degree n. Find the elements and multiplication table of S_3. (8)

(ii) Find the left cosets of $\{[0], [2]\}$ in the group $(Z_4, +_4)$. (4)

(iii) Define a Ring and give an example. (4)

15. (a) (i) Show that in a complemented lattice $(L, \leq)$, $a \leq b \Leftrightarrow a' \vee b = 1 \Leftrightarrow a \wedge b' = 0 \Leftrightarrow b' \leq a'$. (9)

(ii) In a Boolean algebra of all divisors of 70, find all subalgebras. (7)

Or

(b) (i) Show that there are only five distinct Hasse diagram for partially ordered sets that contain three elements. (6)

(ii) Prove the following are equivalent in a Boolean algebra. (10)

(1) $a + b = b$

(2) $a * b = a$

(3) $a' + b = 1$

(4) $a * b' = 0$.
