K 1366

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2004.

Third Semester

Computer Science and Engineering

CS 231 — INTRODUCTION TO ANALYSIS OF ALGORITHMS

Time : Three hours   Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Prove that there are at most 2^q nodes at level q of a binary tree, where the root is assumed to be at level 0.

2. Give two criteria based on which one algorithm is chosen among several available for a given problem.

3. Find the order of $n^4 + 4n + 1$.

4. Write an algorithm that finds the product of two matrices.

5. What is an internal sort algorithm? Give two examples.

6. List the basic operations of an external sort algorithm.

7. Define a weighted diagraph. Draw one with three vertices.

8. Show the adjacency list representation for your graph of question 7.

9. Draw the KMP pattern matching flow chart for the following pattern : ABBA.

10. Formulate any one 'hard' problem.

PART B — (5 × 16 = 80 marks)

11. (i)   Write an algorithm to search linearly for an element X in an ordered list of 'n' entries. What is its worst-case time requirement? Justify.   (8)

    (ii) Write down the straight forward string matching algorithm.   (8)

12. (a) Write the binary search algorithm and analyze it for its worst-case behaviour. Derive all the intermediate formulae that are used. (16)

Or

(b) (i) Write an algorithm that finds the largest in a list of 'n' entries. What is its worst-case analysis? Is this algorithm optimal? Justify. (6)

(ii) Solve the following recurrence relation :

$$Q(n) = n - 1 + 2Q(n/2) \text{ and } Q(1) = 0.$$

Assume that Q is defined for all powers of 2. (10)

13. (a) Write the complete QUICKSORT algorithm and analyze its average-case behaviour. (16)

Or

(b) Write the external sort algorithm with four tapes and analyze its worst-case behaviour. (16)

14. (a) Given a weighted graph, give and explain the algorithm that outputs the edges in a minimal spanning tree. Explain clearly the data structure used in its implementation. (16)

Or

(b) (i) Give the KMP scan algorithm and arrive at its worst-case analysis. (8)

(ii) How many character comparisons are done by the KMP flowchart construction algorithm for the pattern $P = $ 'A ... AB' (m - 1 A's followed by one B)? (8)

15. (a) Explain the following classes of problems : P, NP and NP complete. State any one classical NP complete problem. (16)

Or

(b) (i) Define a cut vertex of a graph. When is a graph said to be biconnected? Draw a biconnected graph with four vertices. (8)

(ii) Let the nodes of a heap are stored in an array level by level beginning with the root and left to right within each level. Show that the left child of the node in the ith cell is in the $2i^{th}$ cell. (8)

K 1366