

**N 1128**

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2004.

Fourth Semester

Electronics and Communication Engineering

MA 034 — RANDOM PROCESSES

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Event  $A$  and  $B$  are such that  $P(A + B) = \frac{3}{4}$ ,  $P(AB) = \frac{1}{4}$  and  $P(\bar{A}) = \frac{2}{3}$  find  $P(B)$ .
2. Write a note on Chebychev inequality.
3. Give an example for conditional distribution.
4. Distinguish between correlation and regression.
5. Give an example for a continuous time random process.
6. Define a Stationary process.
7. State the properties of an ergodic process.
8. Explain the concept of cross correlation function.
9. What is meant by spectral analysis?
10. Describe a linear system.

PART B — (5 × 16 = 80 marks)

11. State and establish the spectral representation theorem for a wide sense stationary process. Also obtain the covariance function and spectral density function average process. (16)

12. (a) (i) If the MGF of  $X$  is  $(5 - 4e^t)^{-1}$ , find the distribution of  $X$  and  $P(X = 5 \text{ or } 6)$ . (8)
- (ii) If  $X$  has exponential distribution EXPO ( $\lambda$ ), find the distribution of  $Y = e^{-\lambda x}$ . (8)

Or

- (b) (i) The first four moments of a distribution about  $X$  are 1, 4, 10 and 45 respectively. Show that the mean is 5, variance is 3,  $\mu_3 = 0$  and  $\mu_4 = 26$ . (8)
- (ii) Define Gamma distribution and derive the moment generating function. Hence obtain the mean and variance. (8)
13. (a) (i) Suppose the point probability density function (PDF) is given by

$$f(x, y) = \begin{cases} \frac{6}{5}(x + y^2); & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Obtain the marginal PDF of  $X$  and that of  $Y$ . Hence or otherwise find  $P\left[\frac{1}{4} \leq y \leq \frac{3}{4}\right]$ . (8)

- (ii) The joint probability mass function of  $X$  and  $Y$  is given below

$x/y$	-1	+1
0	1/8	3/8
1	2/8	2/8

Find correlation coefficient of  $(X, Y)$ . (8)

Or

- (b) (i) The joint probability mass function (PMF) of  $X$  and  $Y$  is

$P(x, y)$	0	1	2	
X	0	0.1	.04	.02
	1	.08	.20	.06
	2	.06	.14	.30

Compute the marginal PMF  $X$  and of  $Y$ ,  $P[X \leq 1, Y \leq 1]$  and check if  $X$  and  $Y$  are independent. (8)

(ii) Two random variables  $X$  and  $Y$  have the joint density

$$f(x, y) = \begin{cases} 2 - x - y, & 0 < x < 1, 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Show that  $\text{corr}(X, Y) = -1/11$ . (8)

(a) Define a Markov chain (MC). Explain :

(i) How you would clarify the states and identify different classes of a MC. Give an example to each class. (8)

(ii) Explain any two applications of a Binomial Process. (8)

Or

(b) (i) State the postulates of a Poisson process. State its properties and establish the additive property for the Poisson process. (8)

(ii) Write a detailed note on sine-wave process. (8)

(a) Distinguish strict stationarity from wide sense stationarity and establish any two results for a weekly stationary time service involving autocorrelation function. (16)

Or

(b) Write a detailed note on each of the following :

(i) Correlation integrals. (8)

(ii) Linear system with random inputs. (8)