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B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2004.

Second Semester

Computer Science and Engineering

MA 035 — DISCRETE MATHEMATICS ·

Time : Three hours   Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Write the following sentence in a symbolic form :

   "Every one who is healthy can do all kinds of work"

2. Define a functionally complete set. Show that the set $[\wedge, \vee]$ is not a functionally completely set.

3. State Pigeonhole principle.

4. Find the coefficient of x^{10} in $(1 + x^5 + x^{10} + \cdots)^3$.

5. If a and b are the elements of a group $(G, *)$, show that $(a * b)^{-1} = b^{-1} * a^{-1}$.

6. Show that the intersection of two normal subgroups is a normal subgroup.

7. If $A = \{2, 3\} \subseteq X = \{2, 3, 6, 12, 24, 36\}$ and the relation $\leq$ is such that $x \leq y$ if x divides y, find the least element and greatest element for A.

8. Draw the Hasse diagram of $(X, \leq)$, where X is the set of positive divisors of 45 and the relation $\leq$ is such that $\leq = \{(x, y): x \in A, y \in A \wedge (x \text{ divides } y)\}$.

9. If a graph has n vertices and a vertex u is connected to a vertex v, show that there is a path from u to v of length no more than $n - 1$.

10. If a graph contains 21 edges, 3 vertices of degree 4 and other each of degree 3, how many vertices do the graphs has?

PART B — (5 × 16 = 80 marks)

11. Solve the following recurrence relation

   (i) $a_{n+2} - 2a_{n+1} + a_n = 2^n$ with initial conditions $a_0 = 2, a_1 = 1$ by using generating functions. (8)

   (ii) Determine the number of positive integers n, $1 \le n \le 1000$ that are not divisible by 2, 3 or 5. (8)

12. (a) (i) Without constructing the truth table obtain the product-of-sums canonical form of the formula $(\neg P \to R) \wedge (Q \leftrightarrow P)$. Hence find the sum-of-products canonical form. (10)

   (ii) Prove that $P \to Q, Q \to R, P \vee R \Rightarrow R$ by using indirect method. (6)

Or

   (b) (i) Define tautology and contradiction. Test whether the following formula : $Q \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$ is a tautology or contradiction without constructing the truth table. (8)

   (ii) Derive $P \to (Q \to S)$ from the premises $P \to (Q \to R), Q \to (R \to S)$ by using the rule CP. (8)

13. (a) (i) If $(G, *)$ is an abelian group and if for all $a, b \in G$, show that $(a * b)^n = a^n * b^n$ for every integer n. (4)

   (ii) State and prove Lagranges Theorem. (12)

Or

   (b) (i) If $(G, *)$ and (H, Δ) are groups and if $f : G \to H$ is a homomorphism, show that the kernel of f is a normal subgroup. (6)

   (ii) Obtain all the elements of S_3. Construct the composition table of S_3 with respect to the operation $\Diamond$. Is $(S_3, \Diamond)$ is abelian? Justify your answer. (10)

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(a) (i) If $(L, *, \oplus)$ is a distributive lattice and if $a * b = a * c$ and $a \oplus b = a \oplus c$ for all $a, b, c \in L$, show that $b = c$ and hence show that complement of an element is unique if it exists in L. (8)

(ii) In a distributive lattice, prove that the following are equivalent :

(1) $a \wedge b \leq x \leq a \vee b$.

(2) $x = (a \wedge x) \vee (b \wedge x) \vee (a \wedge b)$. (8)

Or

(b) (i) Show that in every lattice distributive inequality is true. Further show that every distributive lattice is modular. Whether the converse is true? Justify your claim. (10)

(ii) If a, b, are any two elements in a complemented distributive lattice L, show that the following are equivalent : (6)

$a \leq b, a * b^1 = 0, a^1 \oplus b = 1, b^1 \leq a^1$.

15. (a) (i) Define Eulerian graph and Hamiltonian graph. Give an example of a graph which is Eulerian but not Hamiltonian and vice–versa. (6)

(ii) Show that a nonempty connected graph G is Eulerian if and only if all the vertices of G are of even degree. (10)
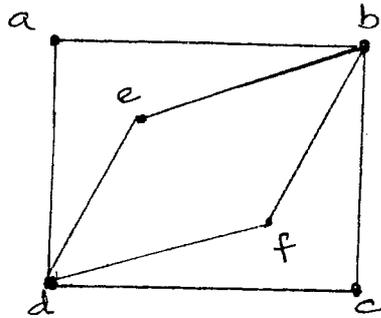
Or

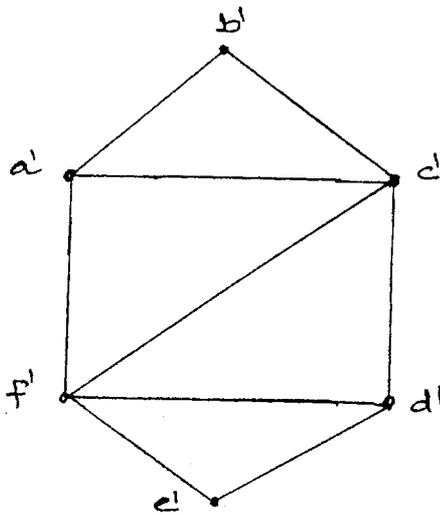(b) (i) Show that in a simple digraph, every node of the digraph lies in exactly one strong component. (6)

(ii) Find the complement of the following graph. (3)
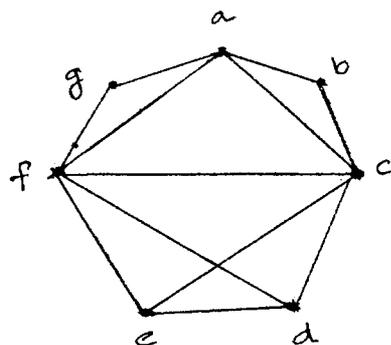
(iii) Determine whether the following graphs G and H are isomorphic. If the graphs are not isomorphic, explain an invariant that the graphs do not share.

(7)

Graph G

Graph H