K 1090

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2004.

Fifth Semester

Computer Science and Engineering

MA 038 — NUMERICAL METHODS

(Common to Metallurgical Engineering and Polymer Technology)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Solve the linear system $4x - 3y = 11$, $3x + 2y = 4$ by Gauss Jordan method.

2. State the condition for convergence of Gauss-Seidel method.

3. If $f(x) = \frac{1}{x^2}$ find the divided differences $f(a,b)$ and $f(a,b,c)$.

4. What is the range of p for which Bessel's formula is better.

5. Given set of values of x and y. If the number of intervals is a multiple of three which formula is used to find the integral of the function.

6. Write the formula for $\frac{dy}{dx}$ at $x = x_0$ using forward difference operator.

7. Solve $\frac{dy}{dx} = x + y$, $y(0) = 1$ to find $y(0.2)$ using Euler's method.

8. Write Milne's Predictor-Corrector formula.

9. Express $a^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$ interms of difference-quotient.

10. Give the Crank-Nicholson difference scheme to solve the parabolic differential equation.

PART B — ($5 \times 16 = 80$ marks)

11. (i) Solve the boundary value problem for $x = 0.5$; $\frac{d^2y}{dx^2} + y + 1 = 0$, $y(0) = y(1) = 0$ [Take $n = 4$]. (6)

(ii) Solve $u_{xx} + u_{yy} = 0$ over the square mesh of side 4 units satisfying the following boundary conditions :

(1) $u(0, y) = 0$ for $0 \le y \le 4$

(2) $u(4, y) = 12 + y$, for $0 \le y \le 4$

(3) $u(x, 0) = 3x$ for $0 \le x \le 4$

(4) $u(x, 4) = x^2$ for $0 \le x \le 4$. (10)

12. (a) (i) Solve

$$\sin xy + x - y = 0$$
$$y \cos xy + 1 = 0$$

given $x_0 = 1, y_0 = 2$ by Newton-Raphson method. (8)

(ii) Using Gauss-Elimination method, solve the system (8)

$$3.15x - 1.96y + 3.85z = 12.95$$
$$2.13x + 5.12y - 2.89z = -8.61$$
$$5.92x + 3.05y + 2.15z = 6.88.$$

Or

(b) (i) Find the largest eigen value and the corresponding eigen vector of the matrix.

$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

using the power method. (8)

(ii) Solve by Jacobi's iteration method, the equations : (8)

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$20z + 2x - 3y = 25.$$

13. (a) (i) Use Stirling formula to find y for $x = 35$ from the following table : (8)

x :	20	30	40	50
y :	512	439	346	243

(ii) From the data given below, find the value of x when $y = 13.5$ by Lagranges inverse interpolation. (8)

x :	93.0	96.2	100.0	104.2	108.7
y :	11.38	12.80	14.70	17.07	19.91

Or

(b) (i) Given the following values of $f(x)$ and $f'(x)$:

x :	–1	0	1
$f(x)$:	1	1	3
$f'(x)$:	–5	1	7

estimate the value of $f(-0.5)$ and $f(0.5)$ using the Hermite interpolation. (10)

(ii) From the following table, evaluate $f(3.8)$ using Newton backward interpolation formula. (6)

x :	0	1	2	3	4
$f(x)$:	1.00	1.50	2.20	3.10	4.60

14. (a) (i) Find the numerical value of the first derivative at $x = 0.4$ of the function $f(x)$ defined as under (8)

x :	0.1	0.2	0.3	0.4
$f(x)$:	1.10517	1.2214	1.34986	1.49182

(ii) Evaluate $\int_0^{\pi/2}\int_0^{\pi/2} \sin(x+y)\,dx\,dy$ by using Simpson's rule. Check the answer by direct integration and obtain the percentage of error. (8)

Or

(b) (i) Evaluate the integral $I = \int_2^4 (x^4 + 1)dx$ using Gaussian quadrature three point formula. (8)

(ii) Evaluate $\int_0^5 \frac{dx}{4x+5}$ by Trapezoidal rule using || coordinates. (8)

15. (a) (i) Using Taylor's series method, obtain the value of y at $x = 0.2$ correct to four decimal places, if y satisfies the equation $\frac{d^2y}{dx^2} = xy$ given that $\frac{dy}{dx} = 1$ and $y = 1$ when $x = 0$. (10)

(ii) Using Adam's method find $y(0.4)$ given $\frac{dy}{dx} = \frac{1}{2}xy$, $y(0) = 1$, $y(0.1) = 1.01$, $y(0.2) = 1.022$ $y(0.3) = 1.023$. (6)

Or

(b) (i) Given $y'' + xy' + y = 0$, $y(0) = 1$, $y'(0) = 0$ find the value of $y(0.1)$ by using Runge-Kutta method of fourth order. (10)

(ii) Using Modified Euler method find y when $x = 0.1$ given that $y(0) = 1$ and $\frac{dy}{dx} = x^2 + y$. (6)
