N 1085

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2004.

Fourth Semester

Computer Science Engineering

MA 040 — PROBABILITY AND QUEUEING THEORY

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. If the probability that a communication system has high selectivity is 0.54 and the probability that it will have high fidelity is 0.81 and the probability that it will have both is 0.18. Find the probability that a system with high Adelity will have high selectivity.

2. The time (in hours) required to repair a machine is exponentially distributed with parameter $\lambda = 1/2$. What is the probability that a repair takes atleast 10 hours given that its duration exceeds 9 hours?

3. If the joint pdf of the random variables (X, Y) is given by $f(x, y) = kxye^{-(x^2+y^2)}$, $x > 0$, $y > 0$, find the value of k.

4. Show that $COV^2(X, Y) \le Var[X]\, Var[Y]$.

5. Given that the autocorrelation function for a stationary ergodic process with no periodic components is $R(z) = 25 + \dfrac{4}{1+6z^2}$. Find the mean and variance of the process $\{X(t)\}$.

6. What is a Markov chain? When can you say that a Markov chain is Homogeneous?

7. What will be the superposition of n independent Poisson processes with respective average rates $\lambda_1, \lambda_2, \cdots, \lambda_n$?

8. Define instantaneous availability $A(t)$ of a component. How does it differ from reliability of a component?

9. What is the probability that a customer has to wait more than 15 min. to get his service completed in a M|M|1 queueing system, if $\lambda = 6$ per hour and $\mu = 10$ per hour?

10. What is the probability that an arrival to an infinite capacity 3 server Poisson queueing system with $\frac{\lambda}{\mu} = 2$ and $P_0 = \frac{1}{9}$ enters the service without waiting?

PART B — (5 × 16 = 80 marks)

11. (i) A given lot of IC chips contains 2% defective chips. Each chip is tested before delivery. The tester itself is not totally reliable. Probability of tester says the chip is good when it is really good is 0.95 and the probability of tester says chip is defective when it is actually defective is 0.94. If a tested device is indicated to be defective, what is the probability that it is actually defective? (8)

(ii) Find the moment generating function of an exponential random variable and hence find its mean and variance. (8)

12. (a) (i) Given the joint density function, $f(x, y) = cx(x-y)$, $0 < x < 2, -x < y < x$. Evaluate c. Find the marginal densities of X and Y. Find the conditional density of Y given $X = x$. (8)

(ii) Calculate the correlation coefficient for the following heights (in inches) of fathers (X) and their sons (Y). (8)

X:	65	66	67	67	68	69	70	72
Y:	67	68	65	68	72	72	69	71

Or

(b) (i) If the joint pdf of (X, Y) is given by $f(x, y) = x + y, 0 \le x, y \le 1$, find the correlation coefficient between X and Y. (8)

(ii) A distribution with unknown mean μ has variance equal to 1.5. Use central limit theorem to find how large a sample should be taken from the distribution in order that the probability will be atleast 0.95 that the sample mean will be within $\mu \pm 0.5$. (8)

13. (a) (i) Show that the process $X(t) = A\cos\lambda t + B\sin\lambda t$ (where A and B are random variables) is wide sense stationary, if (1) $E(A) = E(B) = 0$ (2) $E(A^2) = E(B^2)$ and $E(AB) = 0$. (8)

(ii) Let X be the random variable which gives the interval between two successive occurrences of a Poisson process with parameter λ. Find out the distribution of X. (8)

Or

(b) (i) Draw the state diagram of a birth–death process. Write down the balance equations and obtain expressions for the steady state probabilities. (8)

(ii) State the postulates of Poisson process. Discuss any two properties of Poisson process. (8)

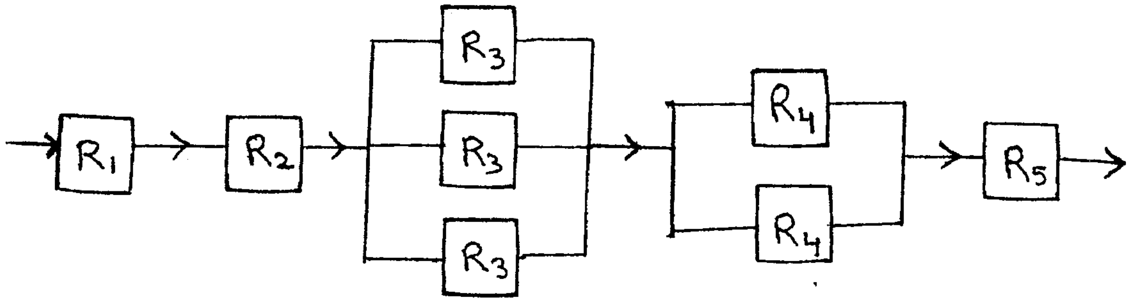
14. (a) (i) The transition probability matrix of a Markov chain $\{X_n\}$ $n = 1, 2, 3, \cdots$ having 3 states 1, 2 and 3 is $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ and the initial distribution is $p^{(0)} = (0.7, 0.2, 0.1)$. Find $P(X_2 = 3, X_1 = 3, X_0 = 2)$. (8)

(ii) Prove that reliability $R(t)$ of a component is given by $e^{-\int_0^t h(x)dx}$ where $h(t)$ is the hazard rate and hence find out the reliability of a component with $h(t) = \lambda_0 t$, where $\lambda_0 > 0$ is a constant. (8)

Or

(b) (i) Find the nature of the states of the Markov chain with the transition probability matrix $\begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix}$. (8)

(ii) What will be the reliability of the system shown in the following figure? Given $R_1 = 0.95$, $R_2 = 0.99$, $R_3 = 0.7$, $R_4 = 0.75$ and $R_5 = 0.9$.



If there is no parallel redundancy, what will be the system reliability? (8)

15. (a) (i) Obtain the expressions for steady state probabilities of a M|M|C queueing system. (8)

(ii) Arrivals at a telephone booth are considered to be Poisson with an average time of 12 min. between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean 4 min. Find the average number of persons waiting in the system. What is the probability that a person arriving at the book will have to wait in the queue? Also estimate the fraction of the day when phone will be in use. (8)

Or

(b) (i) There are three typists in an office. Each typist can type an average of 6 letters per hour. If letters arrive for being typed at the rate of 15 letters per hour, what fraction of time all the typists will be busy? What is the average number of letters waiting to be typed? (8)

(ii) Derive Pollaczek–khintchine formula. (8)