W 2537

M.E. DEGREE EXAMINATION, JANUARY 2007.

First Semester

Computer Aided Design

CD 1602 A — FINITE ELEMENT ANALYSIS

(Common to ME — CAD/CAM and M.E. Engineering Design)

(For candidates admitted during the year 2006–07 onwards)

(Regulation 2005)

me : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Define aspect ratio. State its significance.

2. Classify boundary conditions. Give examples.

3. State the conditions to be satisfied in order to use axisymmetric elements.

4. Sketch a quadratic strain tetrahedran element.

5. What is meant by isoparametric formulation?

6. What is meant by static condensation? State its significance.

7. What is called finite element semidiscretization? Give an example.

8. What are some differences between implicit and explicit methods of numerical integration?

9. Define element capacitance matrix for unsteady state heat transfer problems.

10. Define the stream function for a two – dimensional incompressible flow.

PART B — (5 × 16 = 80 marks)

11. (a) The structure shown in fig.1 is subjected to an increase in temperature of 80° C. Determine the displacements, stresses, and support reactions. Assume the following data

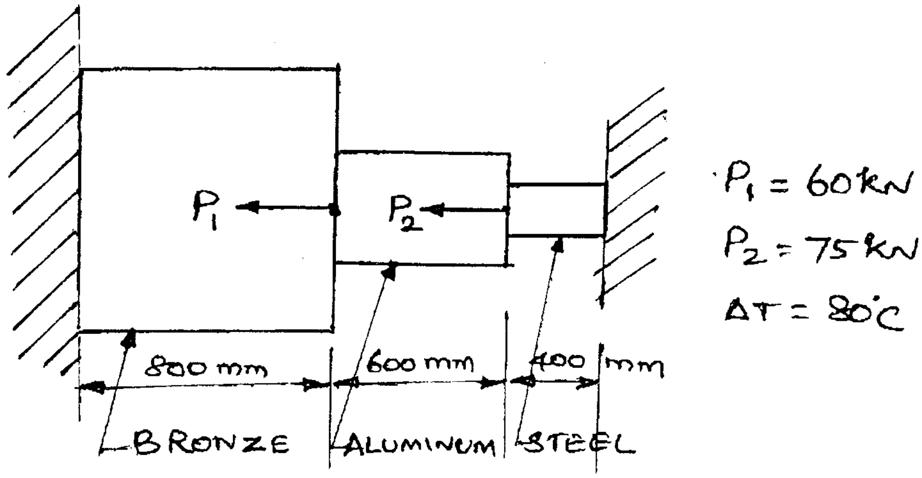

Fig.1

Bronze	Aluminium	Steel
A = 2400 mm^2	1200 mm^2	600 mm^2
E = 83 Gpa	70 Gpa	200 Gpa
α = 18.9 × 10^{-6}/°C	23 × 10^{-6}/°C	11.7 × 10^{-6}/°C

Or

(b) For the three–bar truss shown in Fig.2, determine the displacements of node 1 and the stress in element 3.

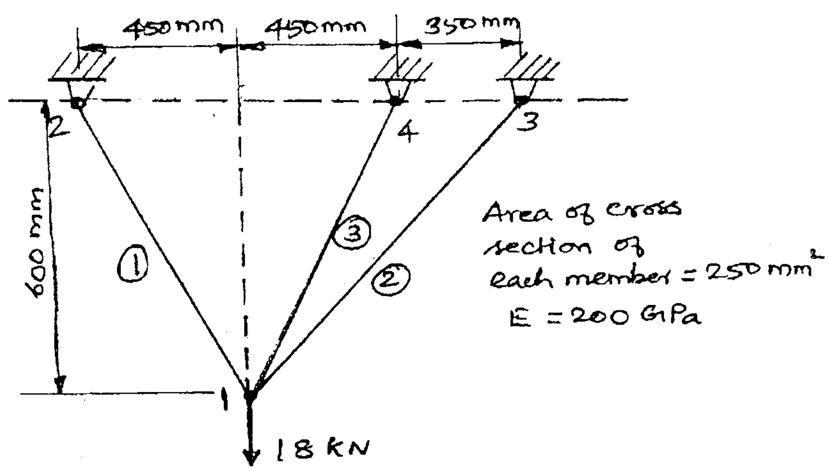

Fig. 2

(a) Evaluate the Conduction matrix, $[K_1^{(e)}]$, for an isotropic rectangular element with four nodes. Use linear temperature variation in x and y directions.

Or

(b) The co–ordinates of the nodes of a three dimensional simplex element are given below, determine the shape functions of the element.

Node Number	Coordinates of the node x	y	z
i	0	0	0
j	10	0	0
k	0	15	0
l	0	0	20

(a) Consider the isoparametric quadrilateral with nodes 1 – 4 at (15, 0), (17, 12) (7, 10), and (6, 2) respectively. Compute the Jacobian matrix and its determinant at the element centroid. Also calculate the area of the element and compare the ratio of the two to the calculated $|J|$. Sketch each element and its parent to scale.

Or

(b) (i) Explain how the stiffness matrix can be obtained for an isoparametric hexahedral element. (10)

(ii) Evaluate the integral $I = \int_{-1}^{1} (a_1 + a_2 x + a_3 x^2 + a_4 x^3)\, dx$ using the three point Gauss integration. (6)

(a) Find the natural frequencies and modes of vibration of two element simply supported beam by taking advantage of the symmetry about the midpoint.

Or

(b) The mass and stiffness matrices for a system are given below :

$$[m] = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.5 \end{bmatrix} \quad [k] = \begin{bmatrix} 3 & -2 & 0 \\ -2 & 4 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

The initial displacement and velocity vectors are given by [0.4, 0.3, 0.3] and [0, 7, 0] respectively. Determine the following :

(i) Natural frequencies (ii) Mode shapes and

(iii) System response.

15. (a) Consider a three – node quadratic conduction / convection element shown in Fig.3 Derive the following for the element :

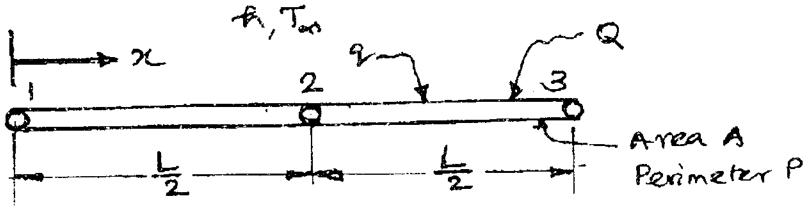

Fig.3

(i) Capacitance matrix (ii) Conduction matrix

(iii) Convection matrix and (iv) Convection heat load vector.

Or

(b) (i) In the finite element analysis of a two – dimensional flow using triangular elements, the velocity components u and v are assumed to vary linearly within an element (e) as

$$u(x,y) = a_1 U_i^{(e)} + a_2 U_j^{(e)} + a_3 U_k^{(e)}$$

$$v(x,y) = a_1 V_i^{(e)} + a_2 V_j^{(e)} + a_3 V_k^{(e)}$$

Where $(U_i^{(e)}, V_i^{(e)})$ denote the values of (u, v) at node i. Find the relationship between $U_i^{(e)}$, $V_i^{(e)}$,......... $V_k^{(e)}$ which is to be satisfied for the flow to be incompressible. (6)

(ii) Explain the potential function formulation of finite element equations for ideal flow problems. (10)