

W 2612

M.E. DEGREE EXAMINATION, JANUARY 2007.

First Semester

Industrial Engineering

IE 1602 — APPLIED OPERATIONS RESEARCH

(Regulation 2005)

Time : Three hours

Maximum : 100 marks

Answer ALL the questions.

PART A — (10 × 2 = 20 marks)

1. How do you deal with the unrestricted variables in the LP model?
2. Why do we need artificial variable while solving LPP?
3. Define shadow price.
4. How do you obtain the solution of the dual from the simplex table of primal?
5. How do you resolve degeneracy in transportation problem?
6. State the main difference between PERT and CPM.
7. What is dynamic programming?
8. Define forward recursive and backward recursive function in Dynamic programming.
9. When is a markov chain said to irreducible?
10. What is balking and reneging in queuing theory?

PART B — (5 × 16 = 80 marks)

11. (a) A firm produces three products. These products are processed on three different machines. The time required to manufacture one unit of each of three products and the daily capacity of the three machines are given in the table below :

Machine	Timer per unit (minutes)			Machine Capacity (minutes/day)
	Product A	Product B	Product C	
M1	2	3	2	440
M2	4	-	3	470
M3	2	5	-	430

It is required to determine the number of units to be manufactured for each product daily. The profit per unit for product 1, 2 and 3 is Rs. 4, Rs. 3 and Rs. 6 respectively. It is assumed that all the amounts produced are consumed in the market.

- (i) Formulate the mathematical model for the problem.
 (ii) Solve the above formulated LPP.

Or

- (b) Joshop needs to assign 4 jobs to 4 workers. The cost of performing a job is a function of the skills of the workers. Following table summarizes the cost of assignments. Worker-1 cannot do job-3, worker-3 cannot do job-4. Determine the optimal assignment using the Hungarian method.

	Job			
	1	2	3	4
Worker 1	50	50	-	20
Worker 2	70	40	20	30
Worker 3	90	30	50	-
Worker 4	70	20	60	70

12. (a) Following is an optimal table of an LPP :

Basic	RHX	X ₁	X ₂	S ₁	A ₁	A ₂	S ₂
X ₁	3/5	1	0	1/5	3/5	-1/5	0
X ₂	6/5	0	1	-3/5	-4/5	3/5	0
S ₂	0	0	0	1	1	-1	1
Z = 12/5	0	0	1/5	M-2/5	M-1/5	0	0

Using sensitivity analysis, find the optimal solution to the new problem obtained by :

- (i) Changing c to $c + c'$ where $c' = (-5, 10)^T$
- (ii) Changing b to $b + b'$ where $b = (3, 4, 1)^T$

Or

- (b) Solve the following integer linear programming problem optimally

$$\text{Maximize } Z = 8x_1 + 6x_2 + 10x_3$$

S.T

$$8x_1 + 4x_2 + 2x_3 \leq 155$$

$$3x_1 + 6x_2 + 12x_3 \leq 135$$

$$x_1, x_2, x_3 \geq 0 \text{ and integers.}$$

- 3. (a) A medium project has twelve distinct activities which are to be further analyzed by using PERT. The following relevant information is also given by tabular form

Activity	Predecessor	Optimistic time (days)	Most likely time (days)	Pessimistic time (days)
A	None	2	2	2
B	None	1	3	7
C	A	4	7	8
D	A	3	5	7
E	B	2	6	9
F	B	5	9	11
G	C,D	3	6	8
H	E	2	6	9
I	C,D	3	5	8
J	G,H	1	3	4
K	F	4	8	11
L	J,K	2	5	7

- (i) Present these activities on PERT network

- (ii) Find the expected float for each activity
- (iii) Determine critical path
- (iv) Within how many days would you expect the project to be completed with 99% chance?

Or

- (b) A manufacturer must produce a certain product in sufficient quantity to meet contracted sales in the next four months. The production facilities available for this product are limited, but by different amounts in respective months. The unit cost of production also varies accordingly to the facilities and personnel available. The product may be produced in one month and then held for sale in a later month, but an estimated storage cost of Re. 1.00 per unit per month. No storage cost is incurred for goods sold in the same month in which they are produced. There is presently no inventory of this product and none is desired at the end of four months. Given the following table show how much to produce in each of four months in order to minimize total cost.

Month	Contracted sales (in units)	Maximum Production (Rs.)	Unit cost of production (Rs.)	Unit storage cost per month (Rs.)
1	20	40	14	1
2	30	50	16	1
3	50	30	15	1
4	40	50	17	1

Formulate the problem as a transportation problem and hence solve it.

14. (a) An electronic item has three components in series. So the reliability of the system is equal to the product of the reliabilities of the three components ($R=r_1 r_2 r_3$). It is a known fact that the reliability of the system can be improved by providing standby units at extra cost. The details of costs and reliabilities for different number of standby units for each of the components of system are summarized in the following table

No. of standby units	Component 1		Component 2		Component 3	
	Cost (Rs.)	Reliability	Cost (Rs.)	Reliability	Cost (Rs.)	Reliability
1	1	0.7	3	0.85	2	0.85
2	2	0.85	4	0.95	3	0.92
3	3	0.95	6	0.98	5	0.97

The total capital budgeted for this purpose is Rs. 8. Determine the optimal number of standby units for each of the components of the system such that the total reliability of the system is maximized.

Or

Solve the following linear programming problem using dynamic programming technique

$$\text{Maximize } Z = 4x_1 + 14x_2$$

S.T

$$2x_1 + 7x_2 \leq 21$$

$$7x_1 + 2x_2 \leq 21$$

$$x_1, x_2, x_3 \geq 0$$

- a) XYZ charitable hospital provides free dental service to the patients on every morning. There are three dentists on duty, who are equally qualified and experienced. It takes, on an average 20 minutes for a patient to get treatment, and the actual time taken is known to vary approximately exponentially around this average. The patients arrive according to the Poisson distribution with an average of 6 per hour. The medical superintendent of the hospital wants to investigate the following.
- (i) The expected number of patients waiting in the queue
 - (ii) The average time that a patient spends at the clinic
 - (iii) The probability that a patient has to wait before he gets service
 - (iv) The average percentage idle time for each of the dentists.

Or

- (b) A business college has three departments : Economics, Accountancy, and Finance, with the following transition probabilities.

	Economics	Accountancy	Finance
Economics	0.30	0.30	0.40
Accountancy	0.01	0.39	0.10
Finance	0.25	0.20	0.55

Interpret the above probabilities. If the initial distributions of students are $[200 \ 500 \ 600]$, find the distribution after two years. Find the long-run probabilities for each department and the long-run distribution of students.
