N 1130

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2004.

Second Semester

Information Technology

MA 039 —PROBABILITY AND STATISTICS

Time : Three hours

Maximum : 100 marks

Statistical Tables may be permitted.

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. In transmitting signals dot and dash, a communication system changes 1/4 of the dots to dashes and 1/3 of the dashes to dots. If 40% of the signals transmitted are dots and 60% are dashes, what is the probability that a dot received was actually a transmitted dot?

2. For a random variable $(RV)X, M_X(t) = \frac{1}{81}\left(e^t + 2\right)^4$. Find $P\{X \le 2\}$.

3. Let X and Y be continuous RVs with joint probability density function (jpdf)

$$f(x, y) = \begin{cases} \frac{3}{2}\left(x^2 + y^2\right) & 0 < x < 1,\ 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find $f_{X|Y}(x \mid y)$.

4. If $E[Y \mid X] = 1$, show that $Var(XY) \ge Var(X)$.

5. Given the random process $\{X(t) = A\cos\varpi_0 t + B\sin\varpi_0 t\}$ $t \in T$, where ϖ_0 is a constant, and A and B are uncorrelated zero–mean random variables having different density functions but the same variances σ^2. Is $\{X(t)\}$ wide–sense stationary?

6. Using the Little's formula obtain the average waiting time in the system for M/M/1/N model.

7. Obtain the reliability function for a RV representing the Weibull failure–time distribution.

8. An engine is to be designed to have a minimum reliability of 0.8 and a minimum availability of 0.98 over a period of 2×10^3 hours. Determine the mean repair time and frequency of failure of the engine.

9. Distinguish between control charts for variables and attributes.

10. What is an essential difference between confidential limits and tolerance limits?

PART B — (5 × 16 = 80 marks)

11. (i) Obtain the availability function for a single component system with repair. Discuss its steady state behaviour. (8)

(ii) For a single component system with the hazard rate $h(t)$ obtain the reliability of the system. (4)

(iii) A system consisting of several identical components connected in parallel is to have a failure rate of atmost 4×10^{-4} per hour. What is the least number of components that must be used if each has a constant failure rate of 9×10^{-4}. (4)

12. (a) (i) The probability density function (pdf) of a continuous RV X is $f_X(x) = \frac{x^n e^{-x}}{n!}$, $x \geq 0$. Show that $P\{0 < X < 2n+2\} > \frac{n}{n+1}$. (6)

(ii) Let X be a continuous RV with set of possible values $\{x : 0 < x < \alpha\}$, (where $\alpha < \infty$) distribution function F and density function f. Prove the following :

$$E[X] = \int_0^{\alpha} [1 - F(t)]dt. \quad (4)$$

(iii) The RV X has an exponential distribution with mean 1. Find the density functions of the variables $Y = 3X + 5$ and $Y = X^2$. (6)

Or

(b) (i) 1200 eggs, of which 200 are rotten, are distributed randomly in 100 cartons each containing a dozen eggs. These cartons are then sold to a restaurant. How many cartons should we expect the chief of the restaurant to open before finding one without rotten eggs? (5)

(ii) First a point Y is selected at random from the interval (0, 1). Then another point X is selected at random from the interval $(Y, 1)$. Find the pdf of X. (6)

(iii) Suppose that jury members decide independently and that each with probability $p\,(0 < p < 1)$ makes the correct decision. If the decision of the majority is final, which is preferable : a 3 person jury or a single juror? (5)

13. (a) (i) If 20 random numbers are selected independently in the interval (0, 1), what is the approximate probability that the sum of these numbers is atleast 8? (6)

(ii) Let X and Y be two independent Uniform RVs over (0, 1); show that the RVs $U = \cos(2\pi X)\sqrt{-2\ln Y}$ and $V = \sin(2\pi X)\sqrt{-2\ln Y}$ are independent standard Normal RVs. (10)

Or

(b) (i) Let (X, Y) have the joint pdf given by :

$$f(x, y) = \begin{cases} 1 & \text{if } \ |y| < x,\ 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Show that the regression of Y on X is linear but regression of X on Y is not linear. (8)

(ii) X and Y are two RVs with variances σ_X^2 and σ_Y^2 respectively and r is the coefficient of correlation between them. If $U = X + kY$ and $V = X + \frac{\sigma_X}{\sigma_Y} Y$, find the value of k so that U and V are uncorrelated.

(8)

14. (a) A salesman's territory consists of 3 cities A, B and C. He never sells in the same city on successive days. If he sells in city A, then the next day he sells in B. However, if he sells either in B or C, then the next day he is twice as likely to sell in city A as in the other city. How often does he sell in each of the cities in the steady state?

Or

(b) (i) State the postulates for a Poisson process. Hence show that the counting process $\{N(t)\}$ follows a Poisson distribution with

$$P_n(t) = \frac{e^{-\lambda t}(\lambda t)^n}{n!} = P(N(t) = n), \quad n = 0, 1, 2, \ldots.$$ (3 + 5)

(ii) If $\{N(t)\}$ is a Poisson process then prove that the auto-correlation coefficient between $N(t)$ and $N(t+s)$ is $\sqrt{\frac{t}{t+s}}$. (8)

15. (a) To find the best arrangement of instruments on a control panel of an airplane, 3 different arrangements were tested by simulating an emergency condition and observing the reaction time required to correct the condition. The reaction times (in lengths of a second) of 28 pilots (randomly assigned to the different arrangements) were as follows :

Arrangement 1 : 14 13 9 15~11 13 14 11

Arrangement 2 : 10 12 9 7~11 8 12 9~10 13 9 10

Arrangement 3 : 11 5 9 10~6 8 8 7 .

Test at the level of significance $\alpha = 0.01$ whether we can reject the null hypothesis that the differences among the arrangements have no effect.

Or

(b) A clothing manufacturer wishes to determine which of 4 different needle designs is best for the sewing machines. The sources of variability that must be eliminated to make this comparison are the actual sewing machine used, the operator, and the type of thread. The manufacturer recorded the number of rejected garments at the end of 2 weeks with the following results :

$D\gamma$ 47	$B\alpha$ 40	$A\beta$ 23	$C\delta$ 72
$C\beta$ 74	$A\delta$ 37	$B\gamma$ 28	$D\alpha$ 75
$A\alpha$ 52	$C\gamma$ 95	$D\delta$ 57	$B\beta$ 15
$B\delta$ 10	$D\beta$ 45	$C\alpha$ 93	$A\gamma$ 52

$C\alpha$ 105	$A\gamma$ 38	$B\delta$ 15	$D\beta$ 60
$D\delta$ 70	$B\beta$ 20	$A\alpha$ 60	$C\gamma$ 85
$B\gamma$ 13	$D\alpha$ 82	$C\beta$ 90	$A\delta$ 28
$A\beta$ 33	$C\delta$ 75	$D\gamma$ 53	$B\alpha$ 31

Using the 0.05 level of significance, determine whether there is a difference in the effectiveness of the needles. Also, determine whether there are significant differences attributable to the operators, the machines, and the types of thread.