Y 5114

E. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2006.

First Semester

Structural Engineering

MA 145 — APPLIED MATHEMATICS

(Regulation 2002)

Three hours Maximum : 100 marks

Normal Table is to be provided.

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. What are the assumptions made while deriving one dimensional wave equation?

2. If $F(\alpha)$ is the Fourier transform of $f(x)$, then find the Fourier transform of $f(x)\cos ax$.

3. Write down any two properties of harmonic functions.

4. What are the various possible solutions of Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$?

5. Write down the simplest variational problem to obtain Euler–Lagrange's equation.

6. Mention Euler–Poisson equation for obtaining stationary value of the functional $I = \int_{x_1}^{x_2} F(x, y, y', y'')dx$.

7. A probability curve $y = f(x)$ has a range from 0 to ∞. If $f(x) = e^{-x}$, find the Mean and Variance.

8. Prove that the correlation coefficient is the geometric mean between regression coefficients.

9. Mention the formula for multiple correlation coefficient $R_{1.23}$.

10. Prove that in sampling from a $N(\mu, \sigma^2)$ population, the sample mean is a consistent estimator of μ.

PART B — (5 × 16 = 80 marks)

11. (a) Solve the boundary value problem in the half-plane $y > 0$ described by the PDE : $u_{xx} + u_{yy} = 0, -\infty < x < \infty, y > 0$, and boundary conditions : $u(x, 0) = f(x), -\infty < x < \infty$, u is bounded as $y \to \infty$; u and $\frac{\partial u}{\partial x}$ both vanish as $|x| \to \infty$. (16)

Or

(b) Using Fourier sine transform, solve the B.V.P.

PDE : $u_{xx} + u_{yy} = 0, \; 0 < x < \infty, 0 < y < a$

BCs : $u(x, 0) = f(x)$

$u(x, a) = 0, u(0, y) = 0, 0 < y < a, 0 < x < \infty$ and $u, \frac{\partial u}{\partial x} \to 0$ as $x \to \infty$. (16)

12. (a) Using Laplace transform method, solve the boundary value problem $u_{xx} = \frac{1}{c^2} u_{tt} - \cos wt, 0 \le x < \infty, 0 \le t < \infty$ and boundary conditions : $u(0, t) = 0$, u is bounded as $x \to \infty$ and initial conditions : $u_t(x,0) = u\,(x, 0) = 0$. (16)

Or

(b) Solve the heat conduction problem described by $k\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, 0 < x < \infty, t > 0$, $u(0, t) = u_0, t \ge 0, u(x, 0) = 0, 0 < x < \infty$. u and $\frac{\partial u}{\partial x}$ both tend to zero as $x \to \infty$. (16)

13. (a) (i) Find the extremal of the functional $V[y(x), z(x)] = \int_0^{\pi/2} (y'^2 + z'^2 + 2yz)\,dx$ given that $y(0) = 0$, $y(\pi/2) = -1$, $z(0) = 0, z(\pi/2) = 1$. (8)

(ii) Determine the extremal of the functional $I[y(x)] = \int_{-a}^{a} \left(\frac{1}{2}\mu y''^2 + \rho y\right) dx$ that satisfies the boundary conditions $y(-a) = 0, y'(-a) = 0, y(a) = 0, y'(a) = 0$. (8)

Or

(i) Write down the Ostrogradsky equation for the functional $I(z(x, y)) = \iint_D \left(\left(\frac{\partial z}{\partial x} \right)^2 + \left(\frac{\partial z}{\partial y} \right)^2 \right) dx\, dy$. (8)

(ii) Find the plane curve of fixed perimeter and maximum area. (8)

(i) In a distribution exactly normal, 7% of the items are under 35 and 89% are under 63. What are the mean and standard deviation of the distribution? (8)

(ii) If the joint distribution function of X and Y is given by $f(x, y) = 1 - e^{-x} - e^{-y} + e^{-(x+y)}$ where $x > 0, y > 0$ and $f(x, y) = 0$, elsewhere

(1) Find the marginal densities of X and Y

(2) Are X and Y independent?

(3) Find $P(X \leq 1 \cap Y \leq 1)$ and $P((X + Y) \leq 1)$. (8)

Or

(b) (i) Compute the coefficient of correlation between X and Y using the following data : (8)

X:	65	67	66	71	67	70	68	69
Y:	67	68	68	70	64	67	72	70

(ii) In a partially destroyed laboratory record of an analysis of correlation data, the following results only are legible. Variance of $X = 1$. The regression equations are $3x + 2y = 26$ and $6x + y = 31$.

What were (1) the mean values of X and Y (2) the standard deviation of Y and (3) the correlation coefficient between X and Y? (8)

15. (a) (i) From the data relating to the yield of dry bark (X_1), height (X_2) and girth (X_3) for 18 cinchona plants the following correlation coefficients were obtained.

$r_{12} = 0.77, r_{13} = 0.72$ and $r_{23} = 0.52$.

Find the partial correlation coefficient $r_{12.3}$ and multiple correlation coefficient $R_{1.23}$. (8)

(ii) Define efficient estimators and prove that sample median is an unbiased and consistent estimator of μ for a normal distribution. (8)

Or

(b) (i) In a trivariate distribution : $\sigma_1 = 2, \sigma_2 = \sigma_3 = 3$, $r_{12} = 0.7$, $r_{23} = r_{31} = 0.5$. Find (1) $b_{12.3}$, $b_{13.2}$ and (2) $\sigma_{1.23}$. (8)

(ii) If $X_1, X_2, \cdots X_n$ are random observations in a Bernoulli variate X taking the value 1 with probability p and the value 0 with probability $(1-p)$, show that $\frac{\Sigma x_i}{n}\left(1-\frac{\Sigma x_i}{n}\right)$ is a consistent estimator of $p(1-p)$. (8)
