Y 5116

M.E. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2006.

First Semester

Thermal Engineering

MA 148 — APPLIED MATHEMATICS FOR MECHANICAL ENGINEERS

(Common to M.E. - Internal Combustion Engineering, M.E. - Energy Engineering, M.E. - Engineering Design, M.E. - Computer Aided Design and M.E. - Refrigeration and Air-conditioning)

(Regulation 2002)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the Laplace transform of unit step function.

2. If u is a function of x and t, find the Fourier cosine transform of $-\frac{\partial^2 u}{\partial x^2}$ in $0 \leq x \geq l$.

3. State the maximum minimum principle of Harmonic function.

4. Find the Fourier Transform of $\frac{\partial u}{\partial t}$.

5. Find the extremal of the functional $\int_{x_0}^{x_1} \frac{y'^3}{x^3}\, dx$.

6. Write the ostrogradsky equation for the functional $V\{z(x, y)\} = \iint_D \{(z_x)^2 - (z_y)^2\}\, dx\, dy$.

7. Derive the five point formula for $\nabla^2 u = 0$.

8. For what value of λ, the explicit method of solving the hyperbolic equation $u_{xx} = \frac{1}{c^2} u_{tt}$ is stable where $\lambda = \frac{c\,\Delta t}{\Delta x}$.

9. State any two basic assumptions in the solution of fluid flow.

10. Find a transformation which maps the hyperbola $x = a\cosh t,\ y = a\sinh t$ into a straight line.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the transformation which maps the semi-infinite strip of the w plane bounded by the lines $v = 0$, $v = \pi$ and $u = 0$ onto the upper half of the z plane, with the points $w = 0$ and $w = i\pi$ mapping on to the points $z = 1$ and $z = -1$ respectively. (8)

(ii) The complex potential of a fluid flow is given by $\Omega(z) = V_0\left[z + \frac{a^2}{z}\right]$ where V_0 and a are positive constants. Obtain equations for the stream lines and equipotential line and the velocity at any point. (8)

Or

(b) (i) Let C be a circle in the z plane having its centre on the real axis, and suppose further that it passes through $z = 1$ and has $z = -1$ as an interior point. Determine the images of C in the W-plane under the transformation $w = f(z) = \frac{1}{2}\left(z + \frac{1}{z}\right)$. (8)

(ii) Find the complex potential due to a source at $z = -a$ and a sink at $z = a$ of equal strength k. Also determine the equipotential lines and stream lines. (8)

12. (a) Using the Laplace Transform method, solve the IBVP described as

$PDE : u_{xx} = \frac{1}{c^2} u_{tt} - \cos wt \quad 0 \le x < \infty, o \le t < \infty$

BCS : $u(0, t) = 0$, u is bounded as x tends to ∞

ICS : $u_t(x, 0) = u(x, 0) = 0$.

Or

(b) Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, x > 0, t > 0.$

subject to the conditions

(i) $u = 0$ when $x = 0, t > 0$

(ii) $u = \begin{cases} 1, & 0 < x < 1 \\ 0, & x \geq 1 \end{cases}$ when $t = 0$ and

(iii) $U(x, t)$ is bounded.

13. (a) Using the method of integral transform, solve the following potential problem in the semi-infinite strip described by

PDE : $u_{xx} + u_{yy} = 0 \quad 0 < x < \infty, \; 0 < y < a$

Subject to

BCs : $u(x, 0) = f(x)$

$u(x, a) = 0$

$u(x, y) = 0 \quad 0 < y < a, 0 < x < \infty$

and $\frac{\partial u}{\partial x}$ tends to zero as $x \to \infty$.

Or

(b) If $\nabla^2 u = 0$, for $x \geq 0$ and if $u = f(y)$ on $x = 0$, show that by using Fourier Transform technique

$$u(x, y) = \frac{x}{\pi} \int_{-\infty}^{\infty} \frac{f(\xi) d\xi}{x^2 + (y - \xi)^2}.$$

14. (a) (i) Find the extremals of the function

$$v[y(x)] = \int_1^2 \frac{\sqrt{1 + y'^2}}{x} \, dx \; ; \; y(1) = 0, y(2) = 1.$$

(ii) Using Ritz method find an approximate solution of the problem of the minimum of the functional

$$V\{y(x)\} = \int_0^2 (y'^2 + y^2 + 2xy) \; dx; \, y(0) = y(2) = 0.$$

Or

(b) (i) Find the extremals of the functional

$$\int_{x_0}^{x_1} (2yz - 2y^2 + y'^2 - z'^2)dx$$

(ii) Find the geodesic on a right circular cone of semi vertical angle α.

15. (a) (i) Derive Bender-Schmidt recurrence equation and discuss its stability.

(ii) Solve $u_{tt} = u_{xx}$ given $u(0,t) = u(4,t) = 0, u(x,0) = \frac{1}{2}x(4-x)$ and $u_t(x,0) = 0$. Take h = 1. Find the solution upto 5 steps in t direction.

Or

(b) (i) Solve $\nabla^2 u = -10(x^2 + y^2 + 10)$ over the square-mesh with sides x = 0, y = 0, x = 3, y = 3 with u = 0 one the boundary and mesh length one unit .

(ii) By Crank Nicholson method, solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to $u(x, 0) = 0$, $u(0,t) = 0$ and $u(1,t) = t$ for two time steps.

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