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B.E DEGREE EXAMINATIONS: MAY/JUNE 2013

Fourth Semester

ELECTRONICS AND COMMUNICATION ENGINEERING

MAT107**:** Random Processes And Vector Spaces

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 1 = 10 Marks)

1. If f(x) is the probability density function then P (a<x<b) is
   a) $\int_{a}^{b} f(x)dx$ b) $\int_{b}^{a2} f(x)dx$
   c) $\int_{-a}^{-b} f(x)dx$ d) $\int_{-b}^{-a} f(x)dx$
2. If the Random Variable X has the following probability function, then the value of k is ----

x	0	1	2	3	4
P(x)	k	3k	5k	7k	9k

   a) 25 b) 30
   c) $\frac{1}{25}$ d) $\frac{1}{30}$
3. If X and Y are independent then, Cov(X,Y) is
   a) 1 b) 0
   c) -1 d) π
4. If the value of the correlation coefficient r is 1 then the correlation is said to be
   a) uncorrelated b) Negative
   c) Positive d) Perfect and positive
5. For a Wide sense stationary process , E[X(t)] is a
   a) Constant b) Function of x
   c) Function of y d) Function of x and y
6. For a Poisson process, Mean is
   a) λ b) t
   c) λt d) λt^2
7. $R_{XY}(\tau)$ is equal to
   a) $R_{YX}(-\tau)$ b) $R_{YX}(\tau)$
   c) $R_{XY}(-\tau)$ d) None
8. If {X(t)} and {Y(t)} are uncorrelated then $C_{XY}(t_1,t_2)$ is
   a) 1 b) -1
   c) 0 d) ∞
9. Let V be a Vector space over a Field F then α0 =
   a) α b) - α
   c) α^2 d) 0

10. The dimension of the subspace spanned by the vectors (1,1,1), (-1,-1,-1) in V_3 (R) is

a) 1 b) 2

c) 0 d) 3

PART B (10 x 2 = 20 Marks)

11. Show that the function

$$f(x) = \begin{cases} \frac{x^2}{3} & -1 < x < 2 \\ 0 \; otherwise \end{cases}$$ is a probability density function.

12. Check whether the following data follows Binomial distribution or not; Mean = 3, Variance = 4

13. Let X and Y have joint density function f(x, y) = 2, 0 < x < y < 1. Find the Marginal density function of X

14. X and Y are independent random variables with variances 2 and 3. Find the variance of 3X+4Y

15. Define: n-step transition probability.

16. Define: Markov Process

17. Find the Mean and Variance of a stationary process whose autocorrelation function is given by $R_{XX}(\tau) = 18 + \{2/(6+ \tau^2)\}$

18. Find the mean square value of the Random Process whose auto correlation function is $(A^2 \cos\omega\tau)/2$

19. Prove that the intersection of two subspaces of a vector space is a subspace.

20. Define: Basis of a vector space.

PART C (5 x 14 = 70 Marks)

21. a) (i) If the density function of a continuous random variable X is given by (7)

$$f(x) = \begin{cases} ax & , 0 \le x \le 1 \\ a & , 1 \le x \le 2 \\ 3a - ax\,, & 2 \le x \le 3 \\ 0 & , otherwise \end{cases}$$ Find the value of a and the cumulative distribution function of X.

(ii) Find the MGF of the Binomial distribution and hence find its mean and variance. (7)

(OR)

b) (i) A random variable X has the following probability distribution (7)

x	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	3k	K^2	$2k^2$	$7k^2+k$

Find the value of k , P(1.5 <X<4.5/X>2), the smallest value of λ for which P(X< λ)>1/2

(ii) Buses arrive at a specified stop at 15 min. intervals starting at 7A.M., that is, they arrive at 7, 7:15, 7:30,and so on. If a passenger arrives at the stop at a random time that is uniformly distributed between 7 and 7:30 A.M., find the probability that he waits i. less than 5 minutes for the bus, ii. At least 12 minutes for a bus. (7)

22. a) The joint probability mass function of (X,Y) is given by p (x,y) = k (2x+3y), x= 0,1,2; y = 1,2,3. Find all the marginal and conditional probability distributions. Also find the probability distribution of X+Y.

(OR)

b) Find the correlation coefficient between X and Y if the joint probability density function of (X,Y) is given by f(x ,y) = 2-x-y 0<x, y <1.

23. a) (i) The process {X(t)} whose probability distribution under certain conditions is given by (7)

$$P[X(t) = n] = \frac{(at)^{n-1}}{(1+at)^{n+1}}, \quad n = 1,2,\ldots$$
$$\frac{at}{1+at}, \quad n = 0$$

Show that it is not stationary.

(ii) Show that the Random process X(t)= A $\cos(\omega_0 t + \theta)$ is wide sense stationary, if A and ω_0 are constants and θ is uniformly distributed Random Variable in (0,2 π). (7)

(OR)

b) (i) A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train but if he drives one day , then the next day he is just as likely to drove again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair die and drove to work if and only if a 6 appeared. Find (7)

1). the probability that he takes a train on the third day and

2). the probability that he drives to work in the long run.

(ii) A machine goes out of order, whenever a component fails. The failure of this part follows a Poisson process with a mean rate of 1 per week. Find the probability that 2 weeks have elapsed since the last failure. If there are 5 spare parts of this component in an inventory and that the next supply is not due in 10 weeks, find the probability that the machine will not be out of order in the next 10 weeks. (7)

24. a) (i) The autocorrelation function of the Poisson increment process is given by (7)

$$R(\tau) = \begin{cases} \lambda^2 & for\ |\tau| > \epsilon \\ \lambda^2 + \frac{\lambda}{\epsilon}\left(1 - \frac{|\tau|}{\epsilon}\right) & for\ |\tau| < \epsilon \end{cases}$$

Find its spectral density.

(ii) If a system is such that its input X(t) and its output Y(t) are related by a convolution integral i.e. if (7)

$$Y(t) = \int_{-\infty}^{\infty} h(u)X(t-u)du$$

Then prove that the system is a linear time-invariant system.

(OR)

b) (i) If the power spectral density of a WSS process is given by (8)

$$S(\omega) = \begin{cases} \frac{b}{a}(a - |\omega|), & |\omega| < a \\ 0, & |\omega| > a \end{cases}$$

Find the autocorrelation function of the process.

(ii) Find the power spectral density of a wide sense stationary process with autocorrelation function $R(\tau) = e^{-\alpha\tau^2}$. (6)

25. a) (i) Prove that RXR is a vector space over R under addition and scalar multiplication defined by (7)

$(x_1, x_2) + (y_1, y_2) = (x_1 + y_1, \quad x_2 + y_2)$ and

$\propto (x_1, x_2) = (\propto x_1, \propto x_2)$

(ii) Prove that $V_n(R)$ is a real inner product space with inner product defined by (7)

$< x, y > = x_1y_1 + x_2y_2 + \ldots + x_ny_n$

where $x = (x_1, x_2, \ldots x_n)$ and $y = (y_1, y_2 \ldots y_n)$.

(OR)

b) Apply Gram Schmidt process to construct an orthonormal basis for $V_3(R)$ with the standard innerproduct for the basis $\{v_1, v_2, v_3\}$ where $v_1 = (1,0,1)$ $v_2 = (1,3,1)$ and $v_3 = (3,2,1)$

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