Register Number: ……………………..

B.E / B.TECH DEGREE EXAMINATIONS: MAY/JUNE 2013

Fifth Semester

MAT108 : NUMERICAL METHODS

(Common to Electronics and Communications Engg and Information Technology)

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 1 = 10 Marks)

1. The order of convergence of Newton-Raphson method is

   a)two b)zero c)one d)1.618

2. In Gauss Jordan method the coefficient matrix is transformed into

   a) an upper triangular matrix b) a lower triangular matrix

   c) a diagonal matrix d) any square matrix

3. If $f(x)=\frac{1}{x}$ then f(a, b) is

   a) $\frac{1}{a\,b}$ b) $\frac{1}{a\,b\,c\,d}$ c) $\frac{a\,b}{b-a}$ d) $-\frac{1}{a\,b}$

4. Newton's backward difference formula is used when x is of

   a) Equal intervals b) unequal intervals

   c) Both equal and unequal interval d) none of the above

5. The formula for $\frac{dy}{dx}\,at\,x=x_n$ using backward difference operator is

   a) $\frac{1}{h}\left[\nabla y_n+\frac{\nabla^2 y_n}{2}+\frac{\nabla^3 y_n}{3}+....\right]$ b) $\left[\nabla y_n+\frac{\nabla^2 y_n}{2}+\frac{\nabla^3 y_n}{3}+.....\right]$

   c) $\frac{1}{h}\left[\nabla y_n-\frac{\nabla^2 y_n}{2}+\frac{\nabla^3 y_n}{3}+....\right]$ d) $\nabla y_n-\frac{\nabla^2 y_n}{2}+\frac{\nabla^3 y_n}{3}+....$

6. The error in the Trapezoidal rule is of the order

   a) h^2 b) h c) h^3 d) $\frac{h}{2}$

7. The method which coincides with the Taylor's series solution up to terms of h^4 is

   a) Runge-Kutta method b) Milne method

   c) Euler method d) Modified Euler Method

8. The Milne's corrector formula is

   a) $y_{n+1}=y_{n-1}-\frac{h}{3}\left(y'_{n-1}+4y'_n+y'_{n+1}\right)$ b) $y_{n+1}=y_{n-1}+\frac{h}{3}\left(y'_{n-1}+4y'_n+y'_{n+1}\right)$

c) $y_{n+1} = y_{n-1} + \frac{h}{3}\left(y'_{n-1} + 4y'_n + y'_{n+1}\right)$ d) $y_{n+1} = y_{n-1} + \frac{h}{3}\left(y'_{n-1} + 4y'_n + y'_{n+1}\right)$

9. The Schmidt's explicit formula for solving heat flow equation is

a) $u_{i,j+1} = \frac{1}{2}\left[u_{i+1,j} + u_{i-1,j}\right]$ b) $u_{i,j+1} = \frac{1}{2}\left[u_{i+1,j} - u_{i-1,j}\right]$

c) $u_{i,j-1} = \frac{1}{2}\left[u_{i+1,j} + u_{i-1,j}\right]$ d) $u_{i,j-1} = \frac{1}{2}\left[u_{i+1,j} - u_{i-1,j}\right]$

10. The Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ is

a) elliptic b) hyperbolic c) parabolic d) None

PART B (10 x 2 = 20 Marks)

11. Solve $3x - y = 2,\ x + 3y = 4$ using Gaussian elimination method.

12. Show that the iterative formula for finding the reciprocal of N is $x_{n+1} = x_n\left(2 - Nx_n\right)$ by Newton's method.

13. Find the third differences of $f(x)$ from the following table.

x	0	1	2	3	4
y = f(x)	1	3	7	13	21

14. Give the Newton's divided difference interpolation formula.

15. Write down the Newton's formula for finding first and second order derivatives at x= x_0.

16. Evaluate $\int_{\frac{1}{2}}^{1} \frac{1}{x}\,dx$ by Trapezoidal rule, dividing the range into 4 equal parts.

17. What do you mean by single step method? Give examples.

18. If $y' = \frac{y - x}{y + x};\ y(0) = 1,$ find $y(0.02)$ by Euler's method.

19. Write down the Bender – Schmidt recurrence relation for one dimensional heat equation.

20. Write down the Crank – Nicholson formula to solve $u_t = u_{xx}$.

PART C (5 x 14 = 70 Marks)

21. a) i) Find a root of $x\log_{10} x - 1.2 = 0$ by false position method correct to 4 decimal places. (7)

ii) Find the inverse of $\begin{pmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{pmatrix}$ by Gauss – Jordan method. (7)

(OR)

b) i) Solve the following system of equations by Gauss-Seidel method correct to four decimal places $28x + 4y - z = 32, x + 3y + 10z = 24, 2x + 17y + 4z = 35$. (7)

ii) Solve the equation $x^3 + x^2 - 1 = 0$ for the positive root by iterative method. (7)

22. a) i) Find the equation $y = f(x)$ of least degree and passing through the points (-1, -21), (1, 15) (2, 12) (3, 3). Find also y at x = 0. (7)

ii) Find the age corresponding to the annuity value 13.6 given the table. (7)

Age (x)	30	35	40	45	50
Annuity value (y)	15.9	14.9	14.1	13.3	12.5

(OR)

b) From the following table of half-yearly premium for policies maturing at different ages, estimate the premium for policies maturing at age 46 and 63.

Age x:	45	50	55	60	65
Premium y:	114.84	96.16	83.32	74.48	68.48

23. a) Find the first two derivatives of $(x)^{1/3}$ at $x = 50$ and $x = 56$ given that table below

x	50	51	52	53	54	55	56
$y = x^{1/3}$	3.6840	3.7084	3.7325	3.7563	3.7798	3.8030	3.8259

(OR)

b) i) Evaluate $I = \int_0^6 \frac{1}{1+x} dx$ by using (a) Trapezoidal rule, (b)Simpson's rule(both) (7)

ii) Evaluate $\int_1^{1.2} \int_1^{1.4} \frac{1}{x+y} dx\, dy$ by Simpson's rule. (7)

24. a) i) Using Runge – Kutta method of fourth order, solve $\frac{dy}{dx}=\frac{y^2-x^2}{y^2+x^2}$ given that $y(0)=1$ at $x=0.2,\ 0.4$ Taking h = 0.2 (7)

ii) Consider the initial value problem $y'=1-y;\ y(0)=0$ (7)

(a) Using the Euler method, find $y(0.1)$.

(b) Using Improved Euler method, find $y(0.2)$.

(OR)

b) Given $\frac{dy}{dx}=xy+y^2,\ y(0)=1,$ find $y(0.1),\ y(0.2)$ & $y(0.3)$ by Taytor's series method and $y(0.4)$ using Milne's predictor-corrector method.

25. a) i) Solve $\frac{\partial^2 u}{\partial x^2}=2\frac{\partial u}{\partial t}$ given $u(0,t)=0,\ u(4,t)=0,\ u(x,0)=x(4-x),$ taking $h=1$. (7)

Find the value of u upto t = 5 using Bender – Schmidt's explicit finite difference
scheme.

ii) Solve $4u_{xx}=u_{tt}$ given $u(0,t)=0;\ u(4,t)=0\ \ u_t(x,0)=0;\ u(x,0)=x(4-x)$ in $0<x<4$. Compute the values of u upto t = 4 by taking h = 1. (7)

(OR)

b) Solve the equation $\nabla^2 u=8x^2y^2$, [h = k = 1] inside a square region bounded by the lines x=±2 and u = 0 on the boundary. Assume the origin at the centre of the square.

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