Register Number: ……..

MCA DEGREE EXAMINATIONS: JUNE/JULY 2013

Second Semester

MASTER OF COMPUTER APPLICATIONS

MAT509: Mathematical Foundations of Computer Science

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 2 = 20 Marks)

1. Find the rank of the matrix $A = \begin{bmatrix} 2 & 3 & 4 & -1 \\ 5 & 2 & 0 & -1 \\ -4 & 5 & 12 & -1 \end{bmatrix}$

2. Two eigen values of $A = \begin{bmatrix} 1 & 0 & 0 \\ 8 & 2 & 0 \\ 6 & 2 & 3 \end{bmatrix}$ are 3 and 2. Find the third eigen value.

3. Define Partition of a set.

4. State the absorption law in set theory.

5. Prove that $P \to (Q \to R) \Leftrightarrow P \to (\neg Q \vee R) \Leftrightarrow (P \wedge Q) \to R$ without using truth table.

6. Using the statements R:Mark is rich, H: Mark is happy. Write the following statements in a symbolic form of "Mark is poor or he is both rich and unhappy"

7. When do you say that the string is accepted by a finite automaton?

8. State Pumping lemma.

9. Define context free grammar.

10. Define Phrase-Structure Grammar.

PART B (5 x 16 = 80 Marks)

11. a) (i) Find for what values of λ and μ the following equations x + y + z = 6; x + 2y + 3z = 10; x + 2y + λz = μ have (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

(ii) Obtain A^{-1} of the matrix $A = \begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix}$ using Cayley Hamilton theorem.

(OR)

b) (i) Obtain the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

(ii) If $\lambda_1, \lambda_2 \ldots\ldots \lambda_n$ are the eigen values of a matrix A then prove that $\frac{1}{\lambda_1}, \frac{1}{\lambda_2} \ldots\ldots \frac{1}{\lambda_n}$ are the eigen values of A^{-1}

12. a) (i) Let R denote a relation on the set of ordered pairs of positive integers such that (x,y) R (u, v) if and only if xv = yu. Show that R is equivalence relation.

(ii) Check whether the function $f(x) = x^2 - 11$ from R to R is one to one, onto or bijective. Justify.

(OR)

b) (i) Prove that $(A \cap B) X (C \cap D) = (AXC) \cap (BXD)$ analytically.

(ii) A survey of 260 television watchers produced the following information: 64 watch sports, 58 watch films, 94 watch news items, 28 watch both sports and films, 26 watch both sports and news items, 22 watch both films and news items and 14 are interested in all the three types. How many people in the survey are not interested in any of these three types? How many people in the survey are interested in watching only news items?

13. a) (i) Find the PDNF of $P \wedge \neg(Q \wedge R) \vee (P \rightarrow Q)$ without using the truth table.

(ii) Show that the following premises are inconsistent

If Rama gets his degree, he will go for a job. If he goes for a job, he will get married soon. If he goes for higher study, he will not get married. Rama gets his degree and goes for higher study.

(OR)

b) (i) Prove that $(\forall x)(P(x) \rightarrow Q(x)) \wedge (\forall x)(Q(x) \rightarrow R(x)) \Rightarrow (\forall x)(P(x) \rightarrow R(x))$

(ii) Obtain the direct proof of $p \rightarrow q, q \rightarrow r, \neg(p \wedge r), p \vee r \Rightarrow r$.

14. a) (i) Explain in detail various types of grammar with example

(ii) Find a phrase structure grammar to generate $\{0^n 1^n 2^n / n = 0,1,2,3\ldots\}$

(OR)

b) (i) Examine whether the following grammar G is ambiguous or not. G= {N,T,S,P} where N= { S,A}, T = {a,b} and P consists of the rules $S \rightarrow aAb$, $S \rightarrow abSb$, $S \rightarrow a$, $A \rightarrow bS$, $A \rightarrow aAAb$.

(ii) Construct a grammar for the language $L = \{a^n b a^m / n, m > 0\}$

15. a) Convert the following NFA to DFA given S_0 is the initial state and S_3 is the final state

States	Inputs	
	a	b
S_0	S_2	S_1
S_1	S_1,S_2	S_3
S_2	ϕ	ϕ
S_3	S_2,S_3	S_2

(OR)

b) The state table of a finite state machine M is given in table.

States	Inputs	
	0	1
S_0	S_2,y	S_1,z
S_1	S_1,x	S_3, y
S_2	S_2, y	S_1,z
S_3	S_3,z	S_0,x

i. Identify the input set I, the state set S, the output set O and the initial state of M

ii. Draw the state diagram of M

iii. Find the output of the string $0^2 1^2 0 1^2 0^2 1$

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