Register Number: ……………………..

MCA DEGREE EXAMINATIONS: JUNE 2013

Second Semester

MASTER OF COMPUTER APPLICATIONS

MAT513: Graph Theory

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 2 = 20 Marks)

1. Define isomorphism with an example.
2. Define directed graph with an example.
3. Define block of a Graph with an example.
4. Define *f*-augmenting path.
5. Find the Mycielski's construction of C_{5}.
6. Explain briefly about the four color conjecture.
7. Define Outerplanar graph with an example
8. State Kuratowski theorem on planar graphs.
9. Explain briefly the circuit matrix with an example
10. Write two observations about a path matrix

PART B (5 x 16 = 80 Marks)

11. a) (i) Prove that a graph is bipartite if and only if it has no odd cycles (10)

(ii) Define King and prove that every tournament is a King (6)

(OR)

b) (i) Prove that the maximum number of edges in an n-vertex triangle free simple graph is $\left\lfloor \frac{n^2}{4} \right\rfloor$ (10)

(ii) Prove that G contains a cycle if G has a vertex of degree at least 2 (6)

12. a) (i) Prove that if G is a simple graph then $\kappa(G) \le \kappa'(G) \le \delta(G)$ (10)

(ii) Explain Ford-Fulkersom labeling algorithm to search an augmenting path to increase the flow value. (6)

(OR)

b) (i) Define connectivity of a graph (2)

(ii) State and prove Mengers theorem (14)

13. a) State and prove Brooks theorem

(OR)

b) (i) Prove that every k-critical graph is $k-1$ edge connected (4)

(ii) Prove that the vertices of every planar graph can be properly colored with five colors. (12)

14. a) (i) Explain Dual graph with an example (4)

(ii) Prove that every 3-connected graph G with at least five vertices has an edge e such that $G.e$ is 3-connected (12)

(OR)

b) (i) State and prove Euler's formula of connected plane graph (8)

(ii) Define Thickness of a graph and prove that and prove that a graph G with n vertices and m edges has thickness at least $m/(3n-6)$ (8)

15. a) (i) Explain Incidence matrix with an example. (6)

(ii) If B and A are respectively the circuit matrix and incidence matrix of a self loop free graph whose columns are arranged using the same order of edges then prove that every row of B is orthogonal to every row of A. (10)

(OR)

b) (i) If B is a circuit matrix of a connected graph G with e edges and n vertices then prove that the rank of B is $e-n+1$ (10)

(ii) What are the observations that can be made about the adjacency matrix of a graph. (6)

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