R 8455

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2006.

Fifth Semester

Computer Science and Engineering

MA 038 — NUMERICAL METHODS

(Common to Metallurgical Engineering/Polymer Technology)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. What is the sufficient condition for the convergence of a root of $f(x) = 0$ by Newton-Raphson method?

2. What is the condition on A for solving the system of equation $AX = B$ by Jacobi and Gauss-Seidel interative methods?

3. Show that the interpolating polynomial for the data.

x :	a	b
$f(x)$:	$f(a)$	$f(2)$

is $P(x) = \dfrac{f(a)(b-x) - f(b)(a-x)}{b-a}$.

4. State the Bessel's central difference formula.

5. What is the error involved in the Trapezoidal rule?

6. Evaluate $I = \int_{-1}^{1} \frac{dx}{1+x^2}$ by one point Gaussian quadrature.

7. Compute y for $x = 0.1$ correct to four decimal places given $y' = y - x$, $y(0) = 2$ using Taylor series method.

8. What is a multistep method?

9. Write down the finite difference scheme for the solution of the Poisson's equation $U_{xx} + U_{yy} = f(x, y)$.

10. Write down the finite difference scheme for the solution of the boundary value problem $y'' + y = 0,\ y(0) = 0,\ y\left(\frac{\pi}{2}\right) = 1$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find a solution of $3x + \sin x - e^x = 0$ correct to four decimal places by Newton-Raphson method. (8)

(ii) Obtain by power method, the numerically largest eigen value of the matrix

$$A = \begin{bmatrix} 15 & -4 & -3 \\ -10 & 12 & -6 \\ -20 & 4 & -2 \end{bmatrix}$$

With the starting vector $X^{(0)} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ perform only 4 interaction. (8)

Or

(b) (i) Using Gauss-Jordan method, find the inverse of the matrix.

$$\begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}$$ (8)

(ii) Apply Gauss-Seidal iteration method to solve the equations 4 decimal accuracy. (8)

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25.$$

12. (a) Find $f(x)$ by Hermite's interpolation from the table :

x :	–1	0	1
f :	1	1	3
f' :	–5	1	7

Compute f and f' at $x = 2$.

Or

(b) (i) Construct Newton's forward interpolation polynomial for the following data : (8)

x :	4	6	8	10
y :	1	3	8	16

(ii) Use Stirling's formula to evaluate $f(1.22)$, given (8)

x :	1.0	1.1	1.2	1.3	1.4
$f(x)$:	0.841	0.891	0.932	0.963	0.985

13. (a) (i) Given the following pairs of values of x and y

x :	1	2	4	8	10
y :	0	1	5	21	27

determine $y'(4)$ using Newton's divided differences. (8)

(ii) Evaluate $\int_1^{1.6} \frac{2x\,dx}{x^2 - 4}$ using the trapezoidal rule with $n = 6$. (8)

Or

(b) Evaluate $\int_0^{0.5}\int_0^{0.5} e^{y-x}\,dx\,dy$ with $\Delta x = \Delta y = 0.125$ using Simpson's $\frac{1}{3}$ rule.

14. (a) (i) Using 4-th order Runge-Kutta method, compute $y(0.2)$ and $y(0.4)$ with $h = 0.2$ from $y' = y - \frac{2x}{y}$, $y(0) = 1$. (8)

(ii) Given $y' = x^2 + y^2$, $y(0) = 1$, $y(-0.1) = 0.9088$, $y(0.1) = 1.1115$, $y(0.2) = 1.2530$ Compute $y(0.3)$ by Milne's method. (8)

Or

(b) Solve $y' = y + 3t - t^2$, $y(0) = 1$ using Euler's method for $y(0.05)$, $y(0.1)$ and $y(0.15)$ and then find $y(0.2)$ by Adam-Bashforth method.

15. (a) Solve by Crank-Micolson's method $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $0 < x < 1, t > 0$, $u(x, 0) = 100x(1 - x)$, $u(0, t) = u(1, t) = 0$ for two time steps with $h = 0.25$ and $k = 0.0625$.

Or

(b) (i) Solve $(x^3+1)y''+x^2y'-4xy=2,\ y(0)=0,\ y(2)=4$ with $h=0.5$. (8)

(ii) Solve $\frac{\partial^2 u}{\partial x^2}=\frac{\partial^2 u}{\partial r^2},\ 0<x<1,\ \ t>0, u(x,0)=100x(1-x), \frac{\partial u}{\partial r}(x,0)=0$,

$u(0,t)=u(1,t)=0$ with $h=k=0.25$ for four time steps. (8)
