C 178

B.E./B.Tech. DEGREE EXAMINATION, JANUARY 2006.

First Semester

(Common to ALL branches of Engineering and Technology)

MA 1101 — MATHEMATICS — I

(Regulation 2004)

Time : Three hours — Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. What is the rank of $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & -2 & -3 \\ 2 & 4 & 6 \end{bmatrix}$?

2. If A is an orthogonal matrix prove that $|A| = \pm 1$.

3. Prove, by direction ratios, the points $(1, 2, 3)$; $(4, 0, 4)$; $(-2, 4, 2)$ are collinear.

4. Write down the equation of the sphere whose diameter is the line joining (1, 1, 1) and (–1,–1,–1).

5. What is the curvature of $x^2 + y^2 - 4x - 6y + 10 = 0$ at any point on it?

6. Find the envelope of the family of straight lines $y = mx \pm \sqrt{m^2 - 1}$, where m is the parameter.

7. If $u = e^x yz^2$ find du.

8. If $x = r\cos\theta$, $y = r\sin\theta$ find $\dfrac{\partial(r, \theta)}{\partial(x, y)}$.

9. Solve : $r\frac{d^2u}{dr^2}+\frac{du}{dr}=0$.

10. Find particular integral of $y''+2y'+5y=e^{-x}\cos 2x$.

PART B — (5 × 16 = 80 marks)

11. (i) Find the evolute of the rectangular hyperbola $xy=c^2$. (10)

(ii) Find the radius of curvature at 't' on $x=e^t\cos t$, $y=e^t\sin t$. (6)

12. (a) (i) Find the particular integral of $y''+7y'-8y=e^{2x}$ by the method of variation of parameters. (10)

(ii) Solve : $y''+2y'+y=x\cos x$. (6)

Or

(b) (i) Solve : $x^2y''-2xy'-4y=x^4$. (6)

(ii) Solve :

$$\frac{dx}{dt}+2x-3y=t$$
$$\frac{dy}{dt}-3x+2y=e^{2t}$$ (10)

13. (a) (i) Expand $f(x,y)=\sin(xy)$ in powers of $(x-1)$ and $(y-\pi/2)$ upto second degree terms. (8)

(ii) If $T=x^3-xy+y^3$, $x=\rho\cos\phi$, $y=\rho\sin\phi$ find $\frac{\partial T}{\partial\rho},\frac{\partial T}{\partial\phi}$. (4)

(iii) If $y=f(x+at)+g(x-at)$, show that $\frac{\partial^2y}{\partial t^2}=a^2\frac{\partial^2y}{\partial x^2}$ where a is constant. (4)

Or

(b) (i) Evaluate $\int_0^\infty e^{-\alpha x} \frac{\sin x}{x} dx$, where $\alpha \geq 0$ and hence show that $\int_0^\infty e^{-x} \frac{\sin x}{x} dx = \frac{\pi}{4}$. (6)

(ii) Find the shortest distance from the origin to the curve $x^2 + 8xy + 7y^2 = 225$. (10)

14. (a) (i) Show that the lines

$$\frac{x+3}{2} = \frac{y+5}{3} = \frac{z-7}{-3} \text{ and } \frac{x+1}{4} = \frac{y+1}{5} = \frac{z+1}{-1}$$

are co-planar and find the equation of the plane containing them. (10)

(ii) Find the equation of the plane through the point (–1, 3, 2) and perpendicular to the planes (6)

$$x + 2y + 2z = 5 \text{ and } 3x + 3y + 2z = 8.$$

Or

(b) (i) Find the centre, radius and area of the circle in which the sphere $x^2 + y^2 + z^2 + 2x - 2y - 4z - 19 = 0$ is cut by the plane $x + 2y + 2z + 7 = 0$. (8)

(ii) Find the two tangent planes to the sphere $x^2 + y^2 + z^2 - 4x + 2y - 6z + 5 = 0$, which are parallel to the plane $2x + 2y = z$. Find their points of contact. (8)

15. (a) (i) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ find A^{-1} and A^3 using Cayley-Hamilton theorem. (6)

(ii) Diagonalize $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ by an orthogonal transformation. (10)

Or

(b) (i) If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, then show that $A^n = A^{n-2} + A^2 - I$ for $n \geq 3$ using Cayley-Hamilton theorem. (6)

(ii) Reduce the quadratic form $q = 2x_1x_2 + 2x_2x_3 + 2x_3x_1$ to canonical form using orthogonal transformation. (10)
