

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2006.

Second Semester

Computer Science and Engineering

MA 035 — DISCRETE MATHEMATICS

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

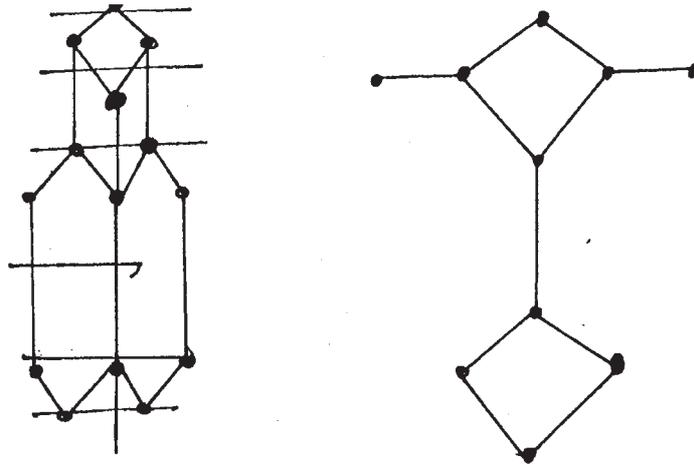
PART A — (10 × 2 = 20 marks)

1. Show that $\{\uparrow\}$ is a minimal functionally complete set.
2. Symbolize the statement "Given any positive integer, there is a greater positive integer".
3. In how many ways a football team of eleven players can be chosen out of 18 players when (a) four particular players are to be always included (b) three particular players are to be always excluded.
4. Find the number of arrangements of the letters in DISCRETE. How many of these arrangements have no adjacent E's?
5. Show that if every element in a group is its own inverse, then the group must be abelian.
6. When an element in a ring is said to be zero divisor? Give an example of a ring without zero divisors.
7. Obtain the partial ordering represented by the Hasse diagram.



8. Give an example of a lattice which is modular but not distributive.

9. Show that in an undirected graph G , sum of the degrees of all the vertices of G is equal to two times the number of edges of G .
10. Find all the cut-vertices and cut-edges of the following graph.



PART B — (5 × 16 = 80 marks)

11. (i) Let G be a simple undirected graph with adjacency matrix A with respect to the ordering v_1, v_2, \dots, v_n . Show that number of different paths of length r from v_i to v_j , where r is a positive integer, equal to the $(i, j)^{th}$ entry of A^r . (8)
- (ii) Show that K_n has a Hamilton circuit for $n \geq 3$. Obtain all the edge disjoint Hamilton circuits of K_7 . (8)
12. (a) (i) Obtain pcnf and pdnf of the formula
 $(\neg P \vee \neg Q) \rightarrow (P \leftrightarrow \neg Q)$. (10)
- (ii) Show that the following sets of premises are inconsistent
 $P \rightarrow Q, P \rightarrow R, Q \rightarrow \neg R, P$. (6)
- Or
- (b) (i) Show that the following implication by using indirect method
 $(R \rightarrow \neg Q), R \vee S, S \rightarrow \neg Q, P \rightarrow Q \Rightarrow \neg P$. (8)
- (ii) Show the following implication
 $(x) (P(x) \rightarrow Q(x)), (x) (R(x) \rightarrow \neg Q(x))$
 $\Rightarrow (x) (R(x) \rightarrow \neg P(x))$. (8)

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13. (a) (i) Use mathematical induction to show that $n^3 - n$ is divisible by 3, for $n \in \mathbb{Z}^+$. (4)
- (ii) Show that among any $n + 1$ positive integers not exceeding $2n$ there must be an integer that divides one of the other integers. (4)
- (iii) Solve the recurrence relation $a_n - 6a_{n-1} + 8a_{n-2} = 3^n$ for $n \geq 2$, where $a_0 = 3$, $a_1 = 7$. (8)

Or

- (b) (i) Solve the recurrence relation
 $a_n = 2(a_{n-1} - a_{n-2})$, where $n \geq 2$ and $a_0 = 1$, $a_1 = 2$. (8)
- (ii) Determine the number of positive integers n , $1 \leq n \leq 1000$, that are not divisible by 2, 3 or 5 but are divisible by 7. (8)

14. (a) (i) Obtain all the elements of (S_3, \diamond) and also construct the composition table with respect to the operation \diamond . (8)
- (ii) Show that group homomorphism preserves, identity, inverse and subgroup. (8)

Or

- (b) (i) Show that in a finite group, order of any subgroup divides the order of the group. (12)
- (ii) Let $f : (G, *) \rightarrow (H, \Delta)$ be group homomorphism. Then show that $\text{Ker}(f)$ is a normal subgroup. (4)

- (10)
15. (a) (i) Let $(L, *, \oplus)$ and (M, \wedge, \vee) be two lattices. Then show that $(L \times M, \square, \sqcup)$ is an algebraic lattice, where for $(a, b), (x, y) \in L \times M$,

$$(a, b) \square (x, y) = (a * x, b \wedge y) \text{ and}$$

$$(a, b) \sqcup (x, y) = (a \oplus x, b \vee y). \quad (10)$$

- (8)
- (ii) Show that in distributive lattice cancellation law is true. Is it true that if a distributive lattice is complemented, then we can find atleast one element which has more than two complements? Justify your answer. (6)

Or

- (b) (i) Show that every totally ordered set is a lattice. (6)
- (ii) Show that in a lattice both distributive inequalities and isotone property are true. (6)
- (iii) Show that in a Boolean algebra
 $(a * b)' = a' \oplus b'$ and $(a \oplus b)' = a' * b'$. (4)
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