H 1397

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2006.

Fourth Semester

Aeronautical Engineering

MA 038 — NUMERICAL METHODS

(Common to Automobile Engineering, Civil Engineering, Instrumentation Engineering, Instrumentation and Control Engineering, Mechanical Engineering, Mechatronics Engineering and Production Engineering)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. State the following statement is 'true or false' and justify.

   The convergence in the Gauss-Seidel method is thrice as fast as in Jacobi's method.

2. Show that the iterative formula for finding the reciprocal of N is $x_{n+1} = x_n(2 - Nx_n)$.

3. Write down the Newton's forward difference formula.

4. Distinguish between interpolation and extrapolation.

5. Evaluate $\int_{\frac{1}{2}}^{1} \frac{1}{x} dx$ by Trapezoidal rule, dividing the range into 4 equal parts.

6. When does Simpson's rule give exact result?

7. How many prior values are required to predict the next value in Adam's method?

8. Write down the Euler's algorithm to solve the ordinary differential equation of the first order.

9. Write the finite difference scheme of the differential equation $y'' + 2y = 0$.

10. Derive the five point formula for solving Laplace equation using finite differences.

11. (i) Using Newton-Raphson method, establish the formula $x_{n+1} = \frac{1}{2}\left(x_n + \frac{N}{x_n}\right)$ to calculate the square root of N. Hence find the square root of 5 correct to four places of decimals. (8)

(ii) Find, by Gaussian elimination, the inverse of the matrix

$$\begin{bmatrix} 4 & 1 & 2 \\ 2 & 3 & -1 \\ 1 & -2 & 2 \end{bmatrix}$$ (8)

12. (a) (i) Construct Newton's forward interpolation polynomial for the following data : (8)

x :	4	6	8	10
y :	1	3	8	16

Use it to find the value of y for $x = 5$.

(ii) Given $u_0 = -4$, $u_1 = -2$, $u_4 = 220$, $u_5 = 546$, $u_6 = 1148$, find u_2 and u_3. (8)

Or

(b) (i) Probability distribution function values of a normal distribution are given as follows :

x :	0.2	0.6	1.0	1.4	1.8
$p(x)$:	0.39104	0.33322	0.24197	0.14973	0.07895

Using a suitable interpolation formula, find $p(1.2)$. (8)

(ii) Give the values

x :	14	17	31	35
$f(x)$:	68.7	64.0	44.0	39.1

find the value of $f(x)$ when $x = 27$. (8)

13. (a) (i) From the following table, find the value of x for which y is minimum and find this value of y.

x :	–2	–1	0	1	2	3	4
y :	2	–0.25	0	–0.25	2	15.75	56

(ii) Using the following data, find $f'(5)$.

x :	0	2	3	4	7	9
$f(x)$:	4	26	58	112	466	922

Or

(b) (i) Dividing the range into 10 equal parts, find the approximate value of $\int_0^{\pi} \sin x \, dx$ by trapezoidal rule.

(ii) Evaluate $\int_0^1 \int_0^1 e^{x+y} \, dxdy$ using Simpson's rule.

14\. (a) Solve $\frac{dy}{dx} = 1 - y$ with the initial condition $x = 0, y = 0$, using Euler's algorithm and tabulate the solutions at $x = 0.1, 0.2, 0.3, 0.4$. Using these remits find $y(0.5)$ using Adams-Bashforth predictor and corrector method.

Or

(b) Apply the fourth order Runge-Kutta method, to find an approximate value of y when $x = 0.2$ and $x = 0.4$ given that $y' = x + y$, $y(0) = 1$ with $h = 0.2$.

15\. (a) Solve $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ subject to the following conditions

$$\left.\begin{array}{l} u(0,t) = 0 \\ u(1,t) = 0 \end{array}\right\} \quad t > 0$$

and $$\left.\begin{array}{l} \frac{\partial u}{\partial t}(x,0) = 0 \\ u(x,0) = \sin^3 \pi x \end{array}\right\} \text{ for all } x \text{ in } 0 \le x \le 1,$$

taking $h = \frac{1}{4}$, compute u for 4 time steps.

Or

(b) Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ in $0 < x < 5$; $t > 0$ given that $u(x,0) = 20$, $u(0,t) = 0$, $u(5,t) = 100$, compute u for one time step with h = 1, by Crank-Nicholson method.

———————