H 1399

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2006.

Second Semester

Information Technology

MA 039 — PROBABILITY AND STATISTICS

Time : Three hours Maximum : 100 marks

Statistical Tables Permitted.

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. A continuous random variable X has a p.d.f $f(x) = 3x^2$ $0 \le x \le 1$. Find a such that $P(X \le a) = P(X > a)$.

2. Find the M.G.F of a binomial distribution.

3. If X is uniformly distributed in $(-1, 1)$ find the p.d.f of $y = \sin\left(\frac{\pi x}{2}\right)$.

4. A random variable X has a mean 3 and variance 2. What is the least value of $P\left(|x - 3| < 2\right)$.

5. State any four properties of Poisson process.

6. Define birth and death process.

7. An equipment is required to meet an inherent availability requirement 0.985 and a mean time between failure of 100 hours. What is the permissible mean time to repair?

8. Three lamps are connected in parallel with failure rate each being 0.002 per hour. Find MTBF.

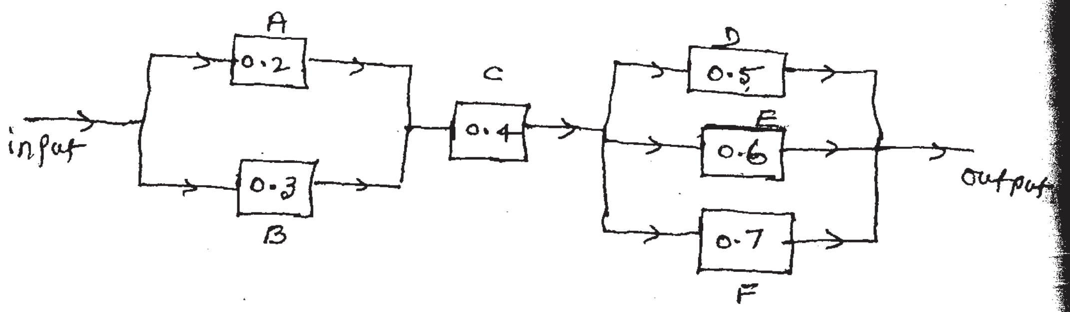
9. Define process control and product control.

10. What are the assumption made in the analysis of variance?

PART B — (5 × 16 = 80 marks)

11. (i) A jet engine has the reliability $R(t) = \frac{1}{1+t}$ $t \geq 0$. How many units be replaced in parallel to get the reliability of 0.97 for 10 years?

(ii) Find the unreliability of the system given below.



12. (a) (i) In a certain recruitment test, there are multiple choice questions. There are 4 possible answers to each question and of which one is correct. An intelligent student knows 90% of the answer. If the intelligent student gets the correct answer what is the probability that he was guessing? (8)

(ii) Find the mean and variance of Weibull distribution. (8)

Or

(b) (i) A and B throw alternately with a pair of dice. A wins if he throws 6 before B throws 7. B wins if he throws 7 before A throw 6. If A begins, find their chances of winning. (8)

(ii) A pair of dice be rolled 900 times and X denotes no. of times a total of 9 occurs. Find $P(80 \leq X \leq 120)$ using Chebyshev inequality. (8)

13. (a) (i) The joint p.d.f of two variables X and Y is

$$f(x, y) = x + y \qquad 0 < x < 1,\ 0 < y < 1$$
$$= 0 \qquad \text{otherwise.}$$

Find the correlation co-efficient between x and y. (8)

(ii) 20 dice are thrown. Find the approximate probability that the sum obtained is between 65 and 75 using central limit theorem. (8)

Or

(b) (i) X and Y are two random variables having the joint density function

$$f(x, y) = 4xy\, e^{-(x^2+y^2)} \quad x, y \geq 0$$

$$= 0 \qquad \text{otherwise.}$$

Find the joint density function of $\sqrt{x^2 + y^2}$. (8)

(ii) Examine whether the variables X and Y are independent, whose joint density is $f(x, y) = x\, e^{-xy-x}$, $0 < x, y < \infty$. (8)

14. (a) (i) Given that WSS random process $X(t) = 10 \cos(100t + \theta)$ where θ is uniformly distributed over $(-\pi, \pi)$. Prove that the process $X(t)$ is correlation-ergodic. (8)

(ii) A TV repairman finds that the time spent on his jobs has an exponential distribution with mean 30 minutes. If he repairs set in the order in which they came in and if the arrival of sets is approximately Poisson with an average rate of 10 per 8 hour day. What is repairman's expected idle time in each day? How many jobs are ahead of the average set just brought in? (8)

Or

(b) (i) The process $\{X(t)\}$ whose probability distribution under certain condition is given by

$$P\{X(t) = n\} = \frac{(at)^{n-1}}{(1+at)^{n+1}} \quad n = 1, 2, 3, \cdots$$

$$= \frac{at}{1+at} \quad n = 0.$$

Show that it is not stationary. (8)

(ii) A repairman is to be hired to repair machines which breakdown at an average rate of 3 per hour. The breakdown follow Poisson distribution. Non-productive time of machine is considered to cost Rs. 16/hour. Two repairman have been interviewed. One is slow but cheap while the other is fast and expensive. The slow repairman charges Rs. 8 per hour and he services machines of a rate of 4 per hour. The fast repairman demands Rs. 10 per hour and services at the average of 6 per hour. Which repairman should be hired? (8)

15. (a) In a Latin square experiment given below, the yields is quintals per on the paddy crop carried out for testing the effect of five fertilizers C, D, E. Analyse the data for variations.

B 25	A 18	E 27	D 30	C 27
A 19	D 31	C 29	E 26	B 23
C 28	B 22	D 33	A 18	E 27
E 28	C 26	A 20	B 25	D 33
D 32	E 25	B 23	C 28	A 20

Or

(b) (i) The following are the sample means and ranges for ten samp each of size 5. Construct the control chart for mean and range a comment on the nature of control. (without drawing the charts)

$[A_2 = 0.577, D_3 = 0, D_4 = 2.115$ for $n = 5]$

Sample No :	1	2	3	4	5	6	7	8	9	10
Mean :	12.8	13.1	13.5	12.9	13.2	14.1	12.1	15.5	13.9	14.2
Range :	2.1	3.1	3.9	2.1	1.9	3.0	2.5	2.8	2.5	2.0

(ii) The data given below are the number of defectives in 10 samples 100 items each. Construct a P chart and comment on the results. (

Sample No :	1	2	3	4	5	6	7	8	9	10
No. of defectives :	6	16	7	3	8	12	7	11	11	4

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