

# H 1401

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2006.

Fourth Semester

Computer Science and Engineering

MA 040 — PROBABILITY AND QUEUEING THEORY

Time : Three hours

Maximum : 100 marks

Statistical Tables are permitted.

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. For a certain binary communication channel, the probability that a transmitted '0' is received as a '0' is 0.95 and the probability that a transmitted '1' is received as a '1' is 0.90. If the probability that a '0' is transmitted is 0.4. Find the probability that a '1' is received.
2. The life length (in months)  $X$  of an electronic component follows an exponential distribution with parameter  $\lambda = \frac{1}{2}$ . What is the probability that the component survives atleast 10 months given that already it had survived for more than 9 months?
3. Define marginal and conditional probabilities of a bivariate probability distribution.
4. Let  $X$  be a continuous random variable with probability distribution  $f(x) = \begin{cases} \frac{x}{12}, & 1 < x < 5 \\ 0, & \text{elsewhere} \end{cases}$ . Find the probability distribution of the random variable  $Y = 2X - 3$ .
5. Let  $X$  be the random variable which gives the inter arrival time (time between successive arrivals), where the arrival process is a Poisson process. What will be the distribution of  $X$ ? How?
6. Define strict sense stationary process and give an example.

7. A man tosses a fair coin until 3 heads occur in a row. Let  $X_n$  denotes the longest string of heads ending at the  $n$ th trial i.e.  $X_n = k$ , if at the  $n$ th trial, the last tail occurred at the  $(n-k)$ th trial. Find the transition probability matrix.
8. What is MTTF? How is it related to reliability function?
9. A duplicating machine maintained for office use is operated by an office assistant. If the jobs arrive at a rate of 5 per hour and the time to complete each job varies according to an exponential distribution with mean 6 minutes, find the percentage of idle time of the machine in a day. Assume that jobs arrive according to a Poisson process.
10. What is the probability that an arrival to an infinite capacity 3 server Poisson queue with  $\frac{\lambda}{c\mu} = \frac{2}{3}$  and  $\rho_0 = \frac{1}{9}$  enters the service without waiting?

PART B — (5 × 16 = 80 marks)

11. (i) Given  $f(x, y) = \begin{cases} cx(x-y), & 0 < x < 2, -x < y < x \\ 0, & \text{elsewhere.} \end{cases}$
- Find
- (1)  $c$
  - (2) The marginal distributions  $f(x)$  and  $f(y)$  and
  - (3) The conditional density of  $Y$  given  $X$   $f(y|x)$ .
- (ii) If  $X$  and  $Y$  each follow an exponential distribution with parameter 1 and are independent, find the pdf of  $U = X - Y$ .
12. (a) (i) The diameter of an electric cable  $X$  is a continuous random variable with pdf  $f(x) = kx(1-x)$ ,  $0 \leq x \leq 1$ . Find (A) the value of  $k$  (B) the cumulative distribution function of  $X$  (C)  $P(X \leq 1/2 | 1/3 < X < 2/3)$ .
- (ii) Find the moment generating function of a Poisson variable and hence obtain its mean and variance.
- Or
- (b) (i) A communication system consists of  $n$  components, each of which will independently function with probability  $p$ . The total system will be able to operate effectively if atleast one-half of its components function. For what values of  $p$ , is a 5-component system more likely to operate effectively than a 3-component system?
- (ii) State and prove Chebyshev's inequality.

13. (a) (i) Check whether the process  $\{X(t)\}$  whose probability distribution is given by

$$P(X(t) = n) = \frac{(at)^{n-1}}{(1+at)^{n+1}}, \quad n = 1, 2, \dots$$

$$= \frac{at}{1+at}, \quad n = 0$$

is stationary.

- (ii) A machine goes out of order, whenever a component fails. The failure of this part follows a Poisson process with a mean rate of 1 per week. Find the probability that 2 weeks have elapsed since last failure. If there are 5 spare parts of this component in an inventory and that the next supply is not due in 10 weeks, find the probability that the machine will not be out of order in the next 10 weeks.

Or

- (b) (i) Draw the state diagram of a birth-death process and obtain the balance equations. Hence, find the limiting distribution of the process.
- (ii) Show that, the renewal function  $M(t)$  of a renewal process is given by  $M(t) = F(t) + \int_0^t M(t-x)f(x)dx$  where  $f(x)$  is the common pdf of  $X_r$ , ( $r = 1, 2, 3, \dots$ ) and  $F(t)$  is the distribution function.
14. (a) (i) The transition probability matrix of a Markov chain  $\{X_n\}_{n=1,2,3,\dots}$

having 3 states 1, 2 and 3 is  $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$  and the initial

distribution is  $P^{(0)} = (0.7, 0.2, 0.1)$ . Find  $P(X_2 = 3)$  and  $P(X_2 = 3, X_1 = 3, X_0 = 2)$ .

- (ii) Prove that the reliability of a system  $R(t) = \exp \left[ - \int_0^t h(x) dx \right]$  where  $h(t)$  is the instantaneous failure rate.

Or

- (b) (i) There are 16 components in a series system. The average reliability of each component is 0.99. In order to achieve atleast this system reliability by using a parallel system with 4 identical new components, what should be the least reliability of each new component?
- (ii) Consider a one unit system with constant failure rate  $\lambda$  and constant repair rate  $\mu$ . Prove that the limiting case availability is given by  $\frac{1/\lambda}{1/\lambda + 1/\mu}$ .

15. (a) (i) Arrivals at a telephone booth are considered to be Poisson with an average time of 12 minutes between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean 4 minutes.
- (A) Find the average number of customers waiting in the system.
  - (B) What is the probability that a person arriving at the booth will have to wait in the queue?
  - (C) Estimate the fraction of the day when the phone will be in use.
  - (D) What is the mean queue length?
- (ii) There are 3 typists in an office. Each typist can type an average of 6 letters per hour. If letters arrive for being typed at the rate of 15 letters per hour.
- (A) What fraction of time all the typists will be busy?
  - (B) What is the average number of letters waiting to be typed?
  - (C) What is the average time a letter has to spend for waiting and for, being typed?

Or

- (b) Derive Pollaczek-Khinchine formula for the average number of customers in the M/G/1 queueing system.