

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2005.

First Semester

Common to all Branches

MA 131 — MATHEMATICS — I

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Prove that eigen values of $-3A^{-1}$ are the same as those of $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$.
2. If the sum of two eigen values and trace of 3×3 matrix A are equal, find the value of $|A|$.
3. Find the equation to the plane bisecting perpendicularly the line joining the points $A(-1, 0, 4)$ and $B(5, 6, -2)$.
4. Show that the lines $\frac{x-1}{2} = \frac{y-1}{4} = \frac{z-1}{0}$ is perpendicular to the z axis.
5. Find the radius of curvature of the curve $y = c \cosh\left(\frac{x}{c}\right)$ at the point where it crosses the y -axis.
6. Find the envelope of the line $\frac{x}{t} + yt = 2c$ where t is parameter.
7. Expand $e^x \log(1+y)$ in powers of x and y upto second degree.
8. If $u = \frac{x+y}{1-xy}$, $v = \tan^{-1} x + \tan^{-1} y$, then prove that u and v are functionally related.
9. Find the particular integral of $(D^2 + 4D + 4)y = xe^{-2x}$.
10. Transform the equation $(2x+3)^2 y'' - 2(2x+3)y' + 2y = 6x$ into a linear equation with constant coefficient.

PART B — (5 × 16 = 80 marks)

11. (i) Find the image of the point (1, 2, -3) in the plane $3x - 3y + 10z = 26$. (6)
- (ii) Find the equation of the sphere described on the line joining the points (2, -1, 4) and (-2, 2, -2) as diameter. Find also the area of the circle in which the sphere is cut by the plane $2x + 2y - z = 3$. (10)

12. (a) (i) Find the eigen values and eigen vectors of the matrix
- $$A = \begin{pmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{pmatrix}. \quad (8)$$

- (ii) Using Cayley-Hamilton theorem, find A^{-1} if $A = \begin{pmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix}$. (8)

Or

- (b) (i) Prove that the product of two orthogonal matrices is an orthogonal matrix. (4)
- (ii) Reduce the quadratic form $2x_1^2 + 6x_2^2 + 2x_3^2 + 8x_1x_3$ to canonical form by orthogonal reduction. (12)
13. (a) (i) If the centre of curvature of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at one end of the minor axis lies at the other end. Prove that the eccentricity of the ellipse is $\frac{1}{\sqrt{2}}$. (7)

- (ii) Prove that the evolute of the curve $x = a \left(\cos \theta + \log \tan \frac{\theta}{2} \right)$, $y = a \sin \theta$ is the catenary $y = a \cosh \left(\frac{x}{a} \right)$. (9)

Or

- (b) (i) Find the envelope of the system of lines $\frac{x}{l} + \frac{y}{m} = 1$ where l and m are parameters connected by the relation $\frac{l}{a} + \frac{m}{b} = 1$. (8)
- (ii) Find the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, treating it as the envelope of its normal. (8)

14. (a) (i) In a triangle ABC , find the maximum value of $\cos A \cos B \cos C$. (8)
- (ii) Find the point on the curve of intersection of the surfaces $z = xy + 5$ and $x + y + z = 1$ which is nearest to the origin. (8)

Or

- (b) (i) If $z = f(u, v)$ where $u = \cosh x \cos y$, $v = \sinh x \sin y$, prove that

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = (\sinh^2 x + \sin^2 y) \left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2} \right). \quad (8)$$

- (ii) Show that $\int_0^{\infty} \frac{\tan^{-1}(ax)}{x(1+x^2)} dx = \frac{\pi}{2} \log(a+1)$. (8)

15. (a) (i) Solve $\frac{dx}{dt} - \frac{dy}{dt} + 2y = \cos 2t$, $\frac{dx}{dt} + \frac{dy}{dt} - 2x = \sin 2t$. (8)

- (ii) Solve $\frac{d^2 y}{dx^2} + y = x \cos x$, by the method of variation of parameter. (8)

Or

- (b) (i) Solve the equation $xy'' - 2(x+1)y' + (x+2)y = (x-2)e^{2x}$ by the method of reduction of order. (8)

- (ii) Solve $(x^2 D^2 - xD + 1)y = \left(\frac{\log x}{x}\right)^2$. (8)