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B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2005.

Second Semester

MA 1151 — MATHEMATICS — II

(Common to all Branches)

(Regulations 2004)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Shade the region of integration $\int_0^a \int_{\sqrt{ax-x^2}}^{\sqrt{a^2-x^2}} dx\, dy$.

2. Express the region $x \geq 0, y \geq 0, z \geq 0$, $x^2 + y^2 + z^2 \leq 1$ by triple integration.

3. If $\nabla \varphi$ is solenoidal find $\nabla^2 \varphi$.

4. Find $\iint_S \bar{r} \cdot \bar{ds}$ where s is the surface of the tetrahedron whose vertices are (0, 0, 0) (1, 0, 0) (0, 1, 0) (0, 0, 1).

5. Find the fixed point of the transformation $w = \frac{6z - 9}{z}$.

6. Give an example such that u and v are harmonic but $u + iv$ is not analytic.

7. Evaluate $\int_c (z^2 + 2z)\, dz$ where c is $|z| = 1$.

8. Expand $\frac{1}{z - 2}$ at $z = 1$ in a Taylors series.

9. Give an example of a function such that it has Laplace transform but it is not satisfying continuity condition.

10. If $L[f(t)] = \frac{s + 2}{s^2 + 4}$. Find the value of $\int_0^\infty f(t)\, dt$.

PART B — (5 × 16 = 80 marks)

11. (i) By converting into polar coordinate evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{x}{x^2+y^2}\, dx\, dy$. (8)

(ii) Evaluate $\iiint \frac{dx\, dy\, dz}{\sqrt{a^2-x^2-y^2-z^2}}$ over the first octant of the sphere $x^2+y^2+z^2=a^2$. (8)

12. (a) (i) Find $\int_c \vec{F}\cdot \overline{d}_r$ where $\overline{F}=(2y+3)\vec{i}+xz\vec{j}+(yz-x)\vec{k}$ along the line joining the points (0, 0, 0) to (2, 1, 1). (8)

(ii) Evaluate $\iint x^3\, dy dz + x^2 y\, dz\, dx + x^2 z\, dx\, dy$ over the surface bounded by $z=0, z=h, x^2+y^2=a^2$. (8)

Or

(b) (i) Prove that $\overline{F}=(y^2\cos x+z^3)\vec{i}+(2y\sin x-4)\vec{j}+3xz^2\vec{k}$ is irrotational and find the scalar potential φ. (7)

(ii) Verify stokes theorem for $\overline{F}=y\,\vec{i}+z\,\vec{j}+x\,\vec{k}$ where s is the upper half of the sphere $x^2+y^2+z^2=1$. (9)

13. (a) (i) If $u=\log(x^2+y^2)$, find V and $f(z)$ such that $f(z)=u+iv$ is analytic. (9)

(ii) Determine the region of the w–plane into which the first quadrant of z–plane mapped by the transformation $w=z^2$. (7)

Or

(b) (i) Construct the analytic function $f(z)=u+iv$ given that $2u+3v=e^x(\cos y-\sin y)$. (8)

(ii) Find the bilinear transformation that maps $z=(1, i, -1)$ into $w=(2, i, -2)$. (8)

14. (a) (i) By Cauchy's integral formula evaluate $\int_c \frac{z+1\, dz}{z^4 - 4z^3 + 4z^2}$ where c is $|z-2-i| = 2$. (8)

(ii) Find the Laurents series expansion of $f(z) = \frac{z}{(z^2+1)(z^2+4)}$ in $1 < |z| < 2$. (8)

Or

(b) (i) By contour integration evaluate $\int_0^{2\pi} \frac{1+2\cos\theta}{5+4\cos\theta} d\theta$. (8)

(ii) By contour integration evaluate $\int_{-\infty}^{\infty} \frac{x^2\, dx}{(x^2+a^2)(x^2+b^2)}$. (8)

15. (a) (i) Find the Laplace transform of the following $e^{2t} \int_0^t \frac{\sin 3t}{t} dt$. (8)

(ii) Using Laplace transform method solve : (8)

$$y'' - 2y' + y = e^t,\ y(0) = 2,\ y'(0) = 1.$$

Or

(b) (i) Find the Laplace transform of $f(t) = \begin{cases} t, & 0 < t < 1 \\ 2-t, & 1 < t < 2 \end{cases}$ such that $f(t+2) = f(t)$. (8)

(ii) Using convolution theorem find the inverse Laplace transform of $\frac{2}{(s+1)(s^2+4)}$. (8)