C 181

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2005.

Third Semester

Civil Engineering

MA 1201 — MATHEMATICS — III

(Regulations 2004)

(Common to All branches of Third Semester Full-time B.E. and Second Semester Part-time B.E.)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Find the PDE of all planes having equal intercepts on the x and y axis.

2. Find the solution of $px^2 + qy^2 = z^2$.

3. Find b_n in the expansion of x^2 as a Fourier Series in $(-\pi, \pi)$.

4. If $f(x)$ is an odd function defined in $(-l, l)$, what are the values of a_o and a_n?

5. What is the basic difference between the solutions of one dimensional wave equation and one dimensional heat equation.

6. In steady state conditions derive the solution of one dimensional heat flow equation.

7. What is the Fourier transform of $f(x-a)$ if the Fourier transform of $f(x)$ is $f(s)$.

8. State the Fourier transforms of the derivatives of a function.

9. Find $z\left[\frac{a^n}{n!}\right]$ in z – transform.

10. Find $z\left[e^{-iat}\right]$ using z – Transform.

PART B — (5 × 16 = 80 marks)

11. (i) Find the Fourier Transform of $f(x)$ if

$$f(x)=\begin{cases}1-|x|, & |x|<1\\ 0 & |x|>1\end{cases}$$ and hence deduce the value of $\int_0^\infty \left[\frac{\sin t}{t}\right]^4 dt$. (8)

(ii) Evaluate $\int_0^\infty \frac{dx}{(x^2+a^2)(x^2+b^2)}$ using Fourier transforms. (8)

12. (a) (i) Solve $(x^2+y^2+yz)p+(x^2+y^2-xz)q=z(x+y)$. (8)

(ii) Solve $(D^2-DD'-20D'^2)z=e^{5x+y}+\sin(4x-y)$. (8)

Or

(b) (i) Solve $z=p^2+q^2$. (8)

(ii) Solve $(D^2+DD'-6D'^2)z=x^2y+e^{3x+y}$. (8)

13. (a) (i) Find the Fourier series for $f(x)=|\cos x|$ in the interval $(-\pi,\pi)$. (10)

(ii) Obtain the half range cosine series for $f(x)=x$ in $(0,\pi)$. (6)

Or

(b) (i) Find the Fourier series for $f(x)=x^2$ in $(-\pi,\pi)$. Hence find $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\frac{1}{4^4}\ldots\ldots\infty$. (8)

(ii) Find the Fourier series up to second harmonic for the following data. (8)

x	:	0	1	2	3	4	5
$f(x)$	:	9	18	24	28	26	20

14. (a) A metal bar 10 cm long with insulated sides, has its ends A and B kept at 20°C and 40°C respectively until steady state conditions prevail. The temperature at A is then suddenly raised to 50°C and at the same instant that at B is lowered to 10°C. Find the subsequent temperature at any point at the bar at any time. (16)

Or

(b) An infinitely long rectangular plate with insulated surface is 10 cm wide. The two long edges and one short edge are kept at zero temperature, while the other short edge $x = 0$ is kept at temperature given by

$u(x, t) = \begin{cases} 20\,y & \text{for} \quad 0 \le y \le 5 \\ 20\,(10 - y) & \text{for} \quad 5 \le y \le 10. \end{cases}$ Find the steady state temperature distribution in the plate. (16)

15. (a) (i) Find $z^{-1}\left[\dfrac{z^3}{(z-1)^2 (z-2)}\right]$ using partial fraction. (8)

(ii) Solve the difference equation $y(k+2) - 4y(k+1) + 4y(k) = 0$ where $y(0) = 1$, $y(1) = 0$. (8)

Or

(b) (i) Prove that $z\left[\dfrac{1}{n+1}\right] = z \log\left(\dfrac{z}{z-1}\right)$. (8)

(ii) State and prove the second shifting theorem in z transform. (8)