B 479

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2005.

Second Semester

MA 132 — MATHEMATICS — II

(Common to all branches except Information Technology)

Time : Three hours Maximum : 100 marks

Use of Statistical Tables permitted.

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Evaluate $\int_0^{\pi/2} \frac{dx}{\sqrt{\cos x}}$.

2. Evaluate $\int_0^{\pi/2} \int_0^{\pi/2} \sin(x+y)\,dx\,dy$.

3. What is the greatest rate of increase of $\phi = xyz^2$ at (1, 0, 3)?

4. Using Green's theorem, find the area of a circle of radius r.

5. Prove that $e^x \cos y$ is a harmonic function.

6. Define bilinear transformation and what is the condition for this to be conformal.

7. State Cauchy's residue theorem.

8. Evaluate $\int_C \frac{dz}{z-a}$ where C is a simple closed curve and a is a point lying inside C.

9. If $\text{var}(x) = 208.69$, $\text{var}(y) = 283.73$, $\text{var}(x-y) = 137.61$, find the coefficient of correlation r.

10. Write down the value of χ^2 for a 2 × 2 contingency table.

PART B — (5 × 16 = 80 marks)

11. (i) Using contour integration evaluate $\int_{-\infty}^{\infty} \frac{x^2\,dx}{(x^2+a^2)(x^2+b^2)}$, $a>0, b>0$. (8)

(ii) Obtain Laurent's series expansion for the function $f(z)=\frac{z^2-1}{z^2+5z+6}$ for (1) $|z|>3$ and (2) $2<|z|<3$. (8)

12. (a) (i) Change the order of integration in $\int_0^a \int_{x^2/a}^{2a-x} xy\,dy\,dx$ and hence evaluate it. (8)

(ii) Evaluate $\iiint xyz\,dx\,dy\,dz$ taken over the positive octant of the sphere $x^2+y^2+z^2=1$. (8)

Or

(b) (i) Evaluate $\int_0^a \int_y^a \frac{x}{x^2+y^2}\,dx\,dy$ by changing into polar coordinates. (8)

(ii) Prove that $\int_0^1 x^m\left(\log\frac{1}{x}\right)^n dx=\frac{\lfloor n+1}{(m+1)^{n+1}}$. (8)

13. (a) (i) Prove that $\nabla\cdot(\vec{u}\times\vec{v})=\vec{v}\cdot curl\,\vec{u}-\vec{u}\cdot curl\,\vec{v}$. Hence or otherwise, prove that if $\vec{u}$ and $\vec{v}$ are irrotational, $\vec{u}\times\vec{v}$ is solenoidal. (8)

(ii) Verify Stoke's theorem for $\vec{F}=-y\hat{i}+2yz\hat{j}+y^2\hat{k}$ where S is the upper half of the sphere $x^2+y^2+z^2=1$. (8)

Or

(b) (i) Prove that $div\,grad\,r^n=n(n+1)r^{n-2}$. Deduce that $div\,grad\,\frac{1}{r}=0$. (8)

(ii) Obtain the expressions for $grad\,\phi$, $div\,\vec{F}$ and $curl\,\vec{F}$ in cylindrical coordinate system. (8)

14. (a) (i) Show that $e^x(x\cos y - y\sin y)$ is a harmonic function. Find the analytic function $f(z)$ for which $e^x(x\cos y - y\sin y)$ is the imaginary part. (8)

(ii) Show that the transformation $w = \frac{1}{z}$ transforms all circles, and straight lines in the z–plane into circles or straight lines in the w–plane. Find which circles in the z–plane become straight lines in the w–plane and which straight lines are transformed into other straight lines. (8)

Or

(b) (i) Derive the necessary conditions for a function $f(z) = u(x, y) + iv(x, y)$ to be analytic at a point in a domain R. (8)

(ii) Find the bilinear transformation which maps the points $\infty, i, 0$ of z–plane into the points $0, i, \infty$ of w–plane. (8)

15. (a) (i) If the density function of a continuous random variable is $f(x) = 6x(1-x), 0 \le x \le 1$ find β_1 and β_2 coefficients. (8)

(ii) Tests made on the breaking strength of 10 pieces of a metal wire gave the results :578, 572, 570, 568, 572, 570, 570, 572, 590 and 584 kg. Test if the mean breaking strength of the wire can be assumed as 577 kg. (8)

Or

(b) (i) Find the coefficient of correlation and lines of regression from the following data : (8)

x :	62	64	65	69	70	71	72	74
y :	126	125	139	145	165	152	180	208

(ii) Two researchers A and B adopted different techniques while rating the students level. Can you say that the technique adopted by them are significant? (8)

Researchers	Below average	Average	Above average	Genius
A	40	33	25	2
B	86	60	44	10