D 023

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2003.

Third Semester

Computer Science and Engineering

CS 231 — INTRODUCTION TO ANALYSIS OF ALGORITHMS

Time : Three hours   Maximum : 100 marks
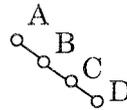
Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Define a height balanced tree. Is the following tree height balanced?

2. Represent the graph in Question 1 using adjacency matrix representation.

3. Prove that a binary tree with depth d has atmost $2^{d+1} - 1$ nodes, where the root is at depth 0.

4. Find the order of $n^2 + \log n$.

5. What input is the "best case" for BUBBLE SORT algorithm? How many comparisons does it do in the best case?

6. Give two applications wherein the algorithm for finding a pattern string in a subject string can be used.

7. Define a minimum spanning tree of a weighted graph. Is it unique for a given graph?

8. What is a cut-vertex of a graph? Identify the cut vertices, if any, of the following graph.
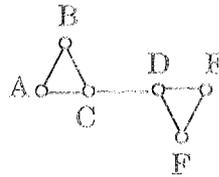
9. State the "Knapsack problem".

10. Define the class NP of problems.

PART B — (5 × 16 = 80 marks)

11. (i) Devise an algorithm that searches sequentially for a specified element in a given list of elements. Do an average-case analysis of the above algorithm. (8)

    (ii) SPLIT algorithm used by QUICK SORT algorithm takes two parameters X, the key and L, a list of entries and rearranges the list such that entries in the list to the left of X are less than or equal to X and entries to the right of X are greater than or equal to X. Note that the key X is an element of L. Devise the algorithm SPLIT. (8)

12. (a) Any algorithm to find the second largest entry in a list of n entries must do atleast $n + \lceil \log n \rceil - 2$ comparisons in the worst case. Prove the above theorem. What is its implication? (16)

Or

    (b) (i) Write an algorithm to find the median of three integers a, b and c. Describe D, the set of inputs of the above algorithm. How many comparisons does your algorithm do in the worst case and in the average case? (10)

        (ii) List the vertices of the graph in question 8 starting from vertex A using depth–first traversal and breadth–first traversal. (6)

13. (a) Write down the complete HEAP SORT algorithm. Do a worst–case analysis of the above algorithm. Derive all the intermediate formulae used. (16)

Or

    (b) (i) Write down the KMP flow chart construction algorithm. Analyze this algorithm for its worst–case behaviour. (10)

        (ii) Draw the KMP pattern matching flowchart for : ABAABB. (6)

14. (a) Write an algorithm which finds a minimal spanning tree, given a weighted graph. Explain clearly the data structure used. Arrive at a formula for the worst-case analysis of the above algorithm. (16)

Or

(b) Given a connected graph represented by linked adjacency lists, write an algorithm that lists the edges in each biconnected component of the graph. Analyze the algorithm for its worst-case behaviour. (16)

15. (a) What are "hard" problems? Discuss clearly, some properties of them. Give 2 examples of hard problems. (16)

Or

(b) Define an approximation algorithm for a problem. State any one classical problem and give an approximation algorithm for it. (16)