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B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2003.

Second Semester

Computer Science and Engineering

MA 035 — DISCRETE MATHEMATICS

Time : Three hours — Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. State the truth value of "If tigers have wings then the earth travels round the sun".

2. Symbolise : For every x, there exist a y such that $x^2 + y^2 \geq 100$.

3. Find the number of unordered samples of size five (repetition allowed) from $\{a, b, c, d, e, f\}$.

4. State the Pigeonhole Principle.

5. Prove that the only idempotent element of a group is its identity element.

6. What do you call a homomorphism of a semi group into itself?

7. Draw the Hasse–diagram of the set of partitions of 5.

8. Prove $a \cdot (a + b) = a + (a \cdot b)$ in a Boolean Algebra.

9. Define a complete graph.

10. What should be the degree of each vertex of a graph G if it has Hamiltonian Circuit?

PART B — ($5 \times 16 = 80$ marks)

11. (i) If $(L, \wedge, \vee)$ is a complemented distributive lattice, prove that the complement $\bar{a}$ of any element $a \in L$ is unique. (8)

(ii) Find the sum–of–product form of the Boolean function $f(x, y, z, w) = xy + y\overline{w}z$. (8)

12. (a) (i) Construct the truth table for

$$(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R).$$ (7)

(ii) Show that

$$(\forall x)(P(x) \vee Q(x)) \Rightarrow (\forall x) P(x) \vee (\exists x) Q(x).$$ (9)

Or

(b) (i) Obtain the principle conjunctive normal form of

$$(\sim P \rightarrow R) \wedge \left(Q \rightleftarrows P\right).$$ (8)

(ii) Using rule CP, derive $P \rightarrow (Q \rightarrow S)$ from $P \rightarrow (Q \rightarrow R)$, $Q \rightarrow (R \rightarrow S)$. (8)

13. (a) (i) Prove by induction : $1 + 2 + 2^2 + \cdots + 2^{n-1} + 2^n = 2^{n+1} - 1$. (6)

(ii) Solve : $S(K) - 4S(K-1) + 4S(K-2) = 3K + 2^K$, $S(0) = 1, S(1) = 1$. (10)

Or

(b) Find the generating function of Fibonacci sequence. (16)

14. (a) (i) Prove that a group G is abelian if and only if $(a * b)^2 = a^2 * b^2$. (8)

(ii) Show that f of the permutation group P_n on to the multiplicative group $G = \{1, -1\}$ defined by

$$f(a) = \begin{cases} 1 & \text{if } a \text{ is even} \\ -1 & \text{if } a \text{ is odd} \end{cases}$$

is a homomorphism. (8)

Or

(b) (i) Prove that in a group G the equations $a * x = b$ and $y * a = b$ have unique solutions for the unknowns x and y as $x = a * b, y = b * a$ where $a, b \in G$. (8)

(ii) Prove that the set of all matrices $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ forms an Abelian group with respect to matrix multiplication. (8)

15. (a) Prove that a given connected graph is Eulerian if and only if all the vertices of G are of even degree.

Or

(b) (i) Write the adjacency matrix of the digraph

$G = \{(V_1, V_3), (V_1, V_2), (V_2, V_4), (V_3, V_1), (V_2, V_3), (V_3, V_4), (V_4, V_1), (V_4, V_2), (V_4, V_3)\}$.

Also draw the graph. (7)

(ii) Prove that a simple graph with n vertices and k components can have atmost $\frac{(n-k)(n-k+1)}{2}$ edges. (9)

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