

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2003.

Fourth Semester

Chemical Engineering

MA 036 — STATISTICS AND LINEAR PROGRAMMING

(Common to Textile, Textile Chemistry and Leather Tech.)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

(Statistical tables may be permitted)

PART A — (10 × 2 = 20 marks)

1. Let X have the probability mass function $p(k) = \begin{cases} \frac{6}{\pi^2} \cdot \frac{1}{k^2}, & k = 1, 2, \dots \\ 0 & \text{otherwise} \end{cases}$. Find the moment generating function.
2. If the probability of success is .09, how many trials are needed to have a probability of atleast one success as $\frac{1}{3}$ or more?
3. Let the joint probability density function of random variables X and Y be given by $f(x, y) = \begin{cases} \frac{1}{2} ye^{-x}, & x > 0, 0 < y < 2 \\ 0 & \text{elsewhere} \end{cases}$. Find the marginal probability density functions of X and Y .
4. Let a random variable X be uniformly distributed over $(-1, 1)$ and $Y = X^2$. Check if X and Y are correlated.
5. What are the usual assumptions made in the analysis of a randomized block experiment?

6. What is the purpose of tolerance limits?
7. What can you say about the optimum of the following problem? Why?

$$\text{Maximize } z = 2x_1 + 4x_2$$

subject to

$$x_1 + 2x_2 \leq 5$$

$$x_1 + x_2 \leq 4$$

$$x_1, x_2 \geq 0.$$

8. True or False : In an LP model, the variable representing the activity with the largest profit per unit in the objective function will always appear at positive level in the optimal solution.

9. Find the optimal objective value of the following problem by inspecting only its dual. (Do not solve the dual by the simplex method)

$$\text{Minimize } z = 10x_1 + 4x_2 + 5x_3$$

subject to

$$5x_1 - 7x_2 + 3x_3 \geq 50$$

$$x_1, x_2, x_3 \geq 0.$$

10. Does the arbitrary selection of the value of one of the multipliers in a transportation iteration lead to the determination of different entering variables?

PART B — (5 × 16 = 80 marks)

11. A company produces two types of cowboy hats. Each hat of the first type requires twice as much labour time as does each hat of the second type. If all hats are of the second type only, the company can produce a total of 500 hats a day. The market limits daily sales of the first and second types to 150 and 200 hats. Assume that the profit per hat is \$8 for type 1 and \$5 for type 2. Determine the number of hats of each type to produce to maximize profit.

12. (a) (i) A toy is rejected if the design is faulty or not. The probability that the design is faulty is 0.1 and that the toy is rejected because of faulty design is 0.95 and otherwise is 0.45. If a toy is rejected, what is the probability that it is due to faulty design? (4)
- (ii) Two unbiased dice are thrown. Let X be a random variable that represents the numbers showing up. Prove by Chebyshev's inequality $P\{|X - 7| \geq 2\} \leq \frac{35}{24}$. Compare this result with the actual probability. (12)

Or

- (b) (i) Let X be the number of births in a hospital until the first girl is born. Determine the probability function and the distribution function of X . Assume the probability that a baby born is a girl is $\frac{1}{2}$. (8)
- (ii) Let the density function of X be $f(x) = \begin{cases} e^{-x} & x > 0 \\ 0 & \text{elsewhere} \end{cases}$. Using the method of transformation, find the density functions of $Y = X\sqrt{X}$ and $Z = e^{-X}$. (8)
13. (a) (i) Customers arrive at a mall in accordance with a Poisson process with rate 4,000 persons per day. Find an approximate value for the probability that tomorrow atleast 3850 customers will enter the mall. (8)
- (ii) Let X and Y be positive independent random variables with the identical probability density function e^{-x} for $x > 0$. Find the joint probability density function of $U = X + Y$ and $V = X | Y$. (8)

Or

- (b) (i) Let the joint probability density function of X and Y be given by

$$f(x, y) = \begin{cases} 6(1 - x - y) & x > 0, y > 0, x + y < 1 \\ 0 & \text{otherwise.} \end{cases}$$

Find $f_X(x)$, $f_Y(y)$ and $Cov(X, Y)$. Are X and Y independent? Obtain the regression curves for the means. (12)

- (ii) Show that for random variables X, Y, Z and W and constants a, b, c and d ,

$$\begin{aligned} \text{Cov}(aX + bY, cZ + dW) &= ac \text{Cov}(X, Z) + bc \text{Cov}(Y, Z) + ad \text{Cov}(X, W) \\ &\quad + bd \text{Cov}(Y, W). \end{aligned} \tag{4}$$

14. (a) A laboratory technician measures the breaking strength of each of 5 kinds of linen threads by using four different measuring instruments, I_1, I_2, I_3 and I_4 and obtains the following results, in ounces :

	I_1	I_2	I_3	I_4
Thread 1	20.9	20.4	19.9	21.9
Thread 2	25.0	26.2	27.0	24.8
Thread 3	25.5	23.1	21.5	24.4
Thread 4	24.8	21.2	23.5	25.7
Thread 5	19.6	21.2	22.1	22.1

Analyze the data using the .05 level of significance.

Or

- (b) An experiment was designed to study the performance of 4 different detergents for cleaning fuel injectors. The following 'cleanness' readings were obtained with specially designed equipment for 12 tanks of gas distributed over 3 different models of engines :

	Engine 1	Engine 2	Engine 3	Totals
Detergent A	45	43	51	139
Detergent B	47	46	52	145
Detergent C	48	50	55	153
Detergent D	42	37	49	128
	182	176	207	565

Perform the ANOVA and test at .01 level of significance whether there are differences in the detergents or in the engines.

15. (a) (i) Solve the following transportation problem : (10)

						Required	
	5	3	7	3	8	5	3
	5	6	12	5	7	11	4
	2	8	3	4	8	2	2
	9	6	10	5	10	9	8
Available	3	3	6	2	1	2	

- (ii) Find an optimal solution to an assignment problem with the following cost matrix : (6)

	J_1	J_2	J_3	J_4
M_1	10	9	7	8
M_2	5	8	7	7
M_3	5	4	6	5
M_4	2	3	4	5

Or

- (b) (i) The following table gives the cost of transporting material from supply points A, B, C and D to demand points, E, F, G, H and J.

	E	F	G	H	J
A	8	10	12	17	15
B	15	13	18	11	9
C	14	20	6	10	13
D	13	19	7	5	12

The present allocation is as follows : A to E 90; A to F 10; B to F 150; C to F 10; C to G 50; C to J 120; D to H 210; D to J 70;

- (1) check if this allocation is optimum. If not, find an optimum schedule. (6)
- (2) If in the above problem the transportation cost from A to G is reduced to 10, what will be the new optimum schedule? (2)

- (ii) Consider the problem of assigning 5 jobs to five persons. The assignment costs are given as follows :

		1	2	3	4	5
Person	A	8	4	2	6	1
	B	0	9	5	5	4
	C	3	8	9	2	6
	D	4	3	1	0	3
	E	9	5	8	9	5

Determine the optimum assignment schedule.

(8)