D 327

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2003.

Second Semester

Information Technology

MA 039 — PROBABILITY AND STATISTICS

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. A and B are events with $P(A)=\frac{3}{8}$, $P(B)=\frac{1}{2}$ and $P(A\cap B)=\frac{1}{4}$. Find $P\left[A^C \cap B^C\right]$.

2. The time (in hours) required to repair a machine is exponentially distributed with parameter $\lambda=\frac{1}{3}$. What is the probability that the repair time exceeds 3 hours.

3. Find the value of K if $f(x, y)=K(1-x)(1-y)$ for $0<x, y<1$ is to be a joint density function.

4. Given the random variable X with density function

$$f(X)=\begin{cases} 2X & 0 < X < 1 \\ 0 & \text{elsewhere} \end{cases}$$

find the probability density function of $Y=8X^3$.

5. When are the processes $\{X(t)\}$ and $\{Y(t)\}$ said to be jointly stationary in the wide sense?

6. Define a Markov Process.

7. Reliability of a component is 0.4. Calculate the number of components to be connected in parallel to get system reliability 0.8.

8. The following data was collected for an automobile :

   Mean time between failures : 500 hr.

   Mean waiting time for spares : 5 hr.

   Mean time for repairs : 48 hr.

   Mean administrative time : 2 hr.

   Compute the availability of the automobile.

9. Name the basic principles of experimental design.

10. Find the lower and upper control limits for $\overline{X}$-chart and s-chart if $n=5$, $\overline{X}=15$ and $\bar{s}=2.5$.

PART B — (5 × 16 = 80 marks)

11. (i) Find the M.G.F. of the random variable X, with probability density function : $f(x)=\begin{cases} x & \text{for } 0\le x<1 \\ 2-x & \text{for } 1\le x<2 \\ 0 & \text{otherwise.}\end{cases}$ Also find μ_1', μ_2'. (8)

(ii) The joint pdf of the two dimensional random variable (X, Y) is given by

$$f(x, y)=\begin{cases} \frac{8}{9}xy & 1\le x\le y\le 2 \\ 0 & \text{elsewhere.}\end{cases}$$

Find the marginal density functions of X and Y.

Find also the conditional density function of Y given $X=x$ and the conditional density function of X given $Y=y$. (8)

12. (a) (i) X is a continuous random variable with pdf given by

$$\begin{aligned} f(x) &= Kx && \text{in } 0\le x\le 2 \\ &= 2K && \text{in } 2\le x\le 4 \\ &= 6K-Kx && \text{in } 4\le x\le 6 \\ &= 0 && \text{elsewhere.}\end{aligned}$$

Find the value of K and also the cdf $f(x)$. (8)

(ii) A random sample of size 100 is taken from a population whose mean is 60 and variance is 400. Using Central Limit Theorem, find with what probability can we assert that the mean of the sample will not differ from $\mu=60$ by more than 4? (8)

Or

(b) (i) Sate Tchebycheff's inequality. Using the inequality for a random variable X with pdf $f(x)=e^{-x}$, $x\ge 0$, show that $P\left[|X-1|>2\right]<\frac{1}{4}$ and show also that the actual probability is e^{-3}. (8)

(ii) Let the random variables X and Y have the joint pdf

$$f(x, y)=\begin{cases} x+y & 0<x<1,\ 0<y<1 \\ 0 & \text{elsewhere.}\end{cases}$$

Compute the correlation coefficient between X and Y. (8)

13. (a) (i) Define Random Process. Specify the four different types of Random Process and give an example to each type. (8)

(ii) The Transition Probability matrix of a Markov chain $\{X_n\}$, $n=0, 1, 2, 3, \cdots$ having 3 states 1, 2 and 3 is $P=\begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ and the initial distribution is $p^{(0)}=(0.7, 0.2, 0.1)$. Find $P\{X_2=3\}$ and $P\{X_3=2, X_2=3, X_1=3, X_0=2\}$. (8)

Or

(b) (i) Prove that the difference of two independent Poisson process is not a Poisson process. (8)

(ii) A Random Process $\{X(t), t\in T\}$ has the probability distribution

$$P\{X(t)=n\}=\frac{(at)^{n-1}}{(1+at)^{n+1}} \quad n=1, 2, \cdots$$
$$=\frac{at}{1+at} \quad n=0.$$

Show that the process is evolutionary. (8)

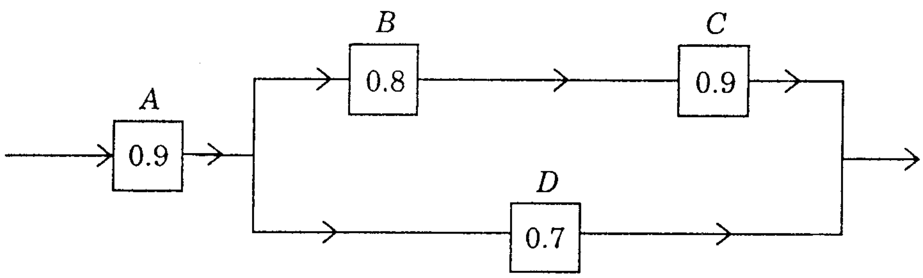
14. (a) (i) The density function of the time to failure of an appliance is $f(t)=\frac{32}{(t+4)^3}$ ($t>0$ is in years). (6)

(1) Find the reliability function $R(t)$

(2) Find the failure rate $\lambda(t)$

(3) Find the MTTF.

(ii) Calculate the system reliability for the units connected as below : (6)

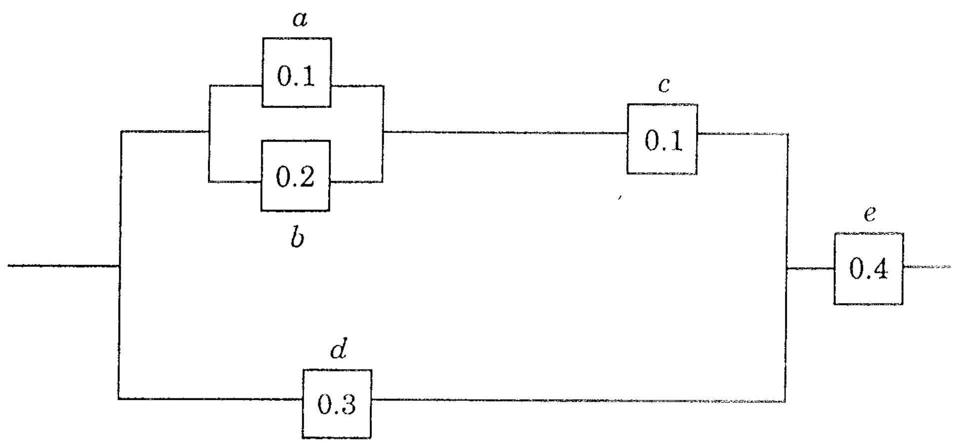


(iii) If a device has a failure rate of $\lambda(t)=(0.015+0.02t)$ where t is in years, calculate the reliability for a 5 year design life, assuming that no maintenance is performed. (4)

Or

(b) (i) Six identical components with constant failure rates are connected in high level redundancy with 3 components in each sub system. Find the component MTTF to provide a system reliability of 0.90 after 100 hours of operation. (6)

(ii) Five elements a, b, c, d and e are connected as shown below. Their reliabilities are also given. (6)



Calculate the system reliability.

(iii) State the relationship between various forms of maintenance. (4)

15. (a) A company appoints 4 salesmen A, B, C and D and observes their sales in 3 seasons : summer, winter and monsoon. The figures (in lakhs of Rs.) are given in the following table :

		Salesmen			
		A	B	C	D
	Summer	45	40	38	37
Season	Winter	43	41	45	38
	Monsoon	39	39	41	41

Carry out an analysis of variance.

Or

(b) The number of customer complaints received daily by an organisation is given below :

Day :	1	2	3	4	5	6	7	8
Complaints :	2	3	0	1	9	2	0	0

Day :	9	10	11	12	13	14	15
Complaints :	4	2	0	7	0	2	4

Does it mean that the number of complaints is under statistical control? Establish a control scheme for the future.