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B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2003.

Second Semester

MA 132 — MATHEMATICS — II

(Common to All Branches except Information Technology)

Time : Three hours Maximum : 100 marks

[Use of Statistical Tables permitted]

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Change the order of integration

$$\int_0^1 \int_{x^2}^{2-x} f(x, y)\, dy\, dx.$$

2. Evaluate using gamma function

$$\int_0^\infty x^6 e^{-3x} dx.$$

3. Find $\nabla \cdot \left(\frac{1}{r}\vec{r}\right)$.

4. For what value of k is the vector $r^k \vec{r}$ solenoidal?

5. Find the analytic region of $f(z) = (x - y)^2 + 2i(x + y)$.

6. What is the image of the line $x = k$ under the transformation $w = \frac{1}{z}$?

7. What is the value of $\int_C e^z dz$, where C is $|z| = 1$?

8. Evaluate $\int_C \frac{\cos \pi z}{z-1} dz$ if C is $|z| = 2$.

9. For the set of numbers 2, 3, 7, 8, 10 find the first four moments.

10. For two variables x and y the two regression equations are $x = 19.13 - 0.87\,y$ and $y = 11.64 - 0.50\,x$, find the correlation coefficient between x and y.

PART B — ($5 \times 16 = 80$ marks)

11. (i) The mean life time of a sample of 100 light bulbs produced by a company is computed to be 1570 hours with a standard deviation of 120 hours. If μ is the mean life time of all the bulbs produced by the company test the hypothesis $\mu = 1600$ hours, against the alternative hypothesis $\mu \neq 1600$ hours with $\alpha = .05$ and .01. (8)

(ii) Carry out a χ^2 test to decide if the proportion of defective packets in each of the following batches is same at .05 level of significance.

Batch	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*
No. of packets :	119	98	150	95	112	60	125	89
No. of faulty packets :	16	28	44	18	30	10	17	18

(8)

12. (a) (i) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx\,dy$ using polar co–ordinates. (8)

(ii) Using the transformation $y^2 = ux, x^2 = vy$ find the area of the region R bounded by the parabolas, $y^2 = ax$, $y^2 = bx, x^2 = cy, x^2 = ey, a > b > 0; c > e > 0$. (8)

Or

(b) (i) Evaluate $\int_{\rho=0}^{1} \int_{z=\rho^2}^{\rho} \int_{\theta=0}^{2\pi} \rho\, d\rho\, dj\, d\theta$. (6)

(ii) Evaluate $\int_0^a y^4 \sqrt{a^2 - y^2}\, dy$ using Beta and Gamma functions. (10)

13. (a) (i) Explain spherical polar co–ordinates using a diagram. (4)

(ii) Find directional derivative of $\phi = 3x^2 + 2y - 3z$ at (1, 1, 1) in the direction $2\hat{i} + 2\hat{j} - \hat{k}$. (4)

(iii) Using Green's theorem in plane evaluate $\oint_C [(y - \sin x)\,dx + \cos x\,dy]$ where C is the triangle OAB where $O = (0, 0)$; $A = (\pi/2, 0)$; $B = (\pi/2, 1)$. (8)

Or

(b) (i) Using divergence theorem evaluate $\int_S \vec{F} \cdot \hat{n}\,dS$ where $\vec{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ and S is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. (6)

(ii) Verify Stokes theorem for $\vec{A} = (2x - y)\,\hat{i} - yz^2\hat{j} - y^2z\hat{k}$ where S is upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. (10)

14. (a) (i) Given that $u = \dfrac{\sin 2x}{\cosh 2y - \cos 2x}$ find the analytic function whose real part is u. (8)

(ii) Find the bi–linear transformation which maps the points $z = 0, 1, \infty$ into $w = -5, -1, 3$ respectively. What are the invariant points of this transformation? (8)

Or

(b) (i) Under the transformation $w = z^2$ find the images of the rectangular hyperbolas $x^2 - y^2 = \lambda$ and $xy = \mu$ where λ and μ are constants and that of the circle $r = a$. (8)

(ii) Draw the image of the square whose vertices are at (0, 0); (1, 0); (1, 1); (0, 1), in the z–plane, under the transformation $w = (1 + i)z$. What has this transformation done to the original square? (8)

15. (a) (i) Expand $\frac{1}{(z-1)(z-2)}$ in the region $|z|>2$ and $0<|z-1|<1$. (8)

(ii) Evaluate $\int_0^{2\pi} \frac{d\theta}{[1-2a\cos\theta+a^2]}, |a|<1$ using Contour integration. (8)

Or

(b) (i) Evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+a^2)(x^2+b^2)} dx, a>0, b>0$ by using Contour integration. (8)

(ii) Form the integral $\int_C \frac{dz}{z+2}$ where C is the circle $|z|=1$, find the value of $\int_0^{2\pi} \frac{1+2\cos\theta}{5+4\cos\theta} d\theta$ and $\int_0^{2\pi} \frac{\sin\theta}{5+4\cos\theta} d\theta$. (8)
