S 9216

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2006.

Fourth Semester

Computer Science and Engineering

MA 040 — PROBABILITY AND QUEUEING THEORY

Time : Three hours   Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If A and B are events with $P(A)=\frac{3}{8}, P(B)=\frac{1}{2}$ and $P(A \cap B)=\frac{1}{4}$, find $P(A^c \cap B^c)$.

2. State Chebyshev Inequality.

3. Find k if the joint probability density function of a bivariate random variable (X, Y) is given by

$$f(x, y)=\begin{cases} k(1-x)(1-y) & \text{if } 0<x, y<1 \\ 0 & \text{otherwise.} \end{cases}$$

4. Two random variable X and Y have joint probability density function

$$f(x, y)=\begin{cases} \frac{xy}{96} & \text{if } 0<x<4, 1<y<5 \\ 0 & \text{otherwise.} \end{cases}$$

Find $E(XY)$.

5. Define strict sense and wide sense stationary process.

6. Show that the sum of two independent Poisson process is a Poisson process.

7. The transition probability matrix of a Markov chain $\{X_n\}$, $n = 1, 2, 3, \cdots$ with three state 0, 1 and 2 is

$$P = \begin{pmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{pmatrix}$$

with initial distribution $P^{(0)} = (1/3, 1/3, 1/3)$. Find $P(X_3 = 1, X_2 = 2, X_1 = 1, X_0 = 2)$.

8. Reliability of the component is 0.4 Calculate the number of component to be connected in parallel to get a system reliability of 0.8.

9. State Little's formula for an (M/M/1) : (∞/FIFO) queueing model.

10. Obtain the steady state probabilities of an (M/M/1) : (*N*/FIFO) queueing model.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Four coins are tossed simultaneously. What is the probability of getting (1) two heads (2) atleast two heads. (4)

(ii) The probability function of an infinite discrete distribution is given by $P(X = j) = 1/2^j$, $j = 1, 2, \cdots \infty$. Find the mean and variance of the distribution. Also find $P(X$ is even), $P(X \geq 5)$ and $P(X$ is divisible by 3). (8)

(iii) The time required to repair a machine is exponentially distributed with parameter 1/2. What is the conditional probability that a repair takes at least 10 hours given that its duration exceeds 9 hours? (4)

Or

(b) (i) *A* and *B* alternatively throw a pair of dice. *A* wins if he throws 6 before *B* throws 7. *B* wins if he throws 7 before *A* throws 6. If *A* begins the game, show that his chances of winning is 30/61. (4)

(ii) Find the mean, variance and moment generating function of a random variable uniformly distributed in the interval (a, b). (8)

(iii) Let X be a continuous random variable with pdf

$$f(x) = \begin{cases} \frac{x}{12}, & 1 < x < 5 \\ 0, & \text{otherwise.} \end{cases}$$

find the probability density function of $2X - 3$. (4)

12. (a) (i) If X and Y are independent random variable with pdf e^{-x}, $x \geq 0$ and e^{-y}, $y \geq 0$, find the density function of $U = \frac{X}{X+Y}$ and $V = X + Y$. Are they independent? (6)

(ii) If the joint pdf of a random variable (X, Y) is given by $f(x, y) = x^2 + \frac{xy}{3}$, $0 \leq x \leq 1, 0 \leq y \leq 2$, find the conditional densities of X given Y and Y given X. (10)

Or

(b) (i) The pdf of X and Y is given by $f(x, y) = k\,x\,y\,e^{-(x^2+y^2)}$, $x > 0, y > 0$. Find k and prove that X and Y are independent. (6)

(ii) Let X and Y be random variables having joint density function

$$f(x, y) = \begin{cases} \frac{3}{2}(x^2 + y^2), & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

Find the correlation coefficient r_{XY}. (10)

13. (a) (i) If $\{N_1(t)\}$ and $\{N_2(t)\}$ are Poisson process with parameter λ_1 and λ_2 respectively. Show that

$\Pr\{N_1(t) = k / N_1(t) + N_2(t) = n\} = {}^nC_k p^k q^{n-k}$, where $p = \frac{\lambda_1}{\lambda_1 + \lambda_2}$ and $q = \frac{\lambda_2}{\lambda_1 + \lambda_2}$. (8)

(ii) The probability distribution of the process $\{X(t)\}$ is given by

$$P(X(t)=n)=\begin{cases}\dfrac{(at)^{n-1}}{(1+at)^{n+1}}, & n=1,2,3,\cdots\\ \dfrac{at}{1+at}, & n=0.\end{cases}$$

Show that it is not stationary. (8)

Or

(b) (i) If the customers arrive in accordance with the Poisson process, with mean rate of 2 per minute, find the probability that the interval between 2 consecutive arrivals is (1) more than 1 minute (2) between 1 and 2 minutes (3) less than 4 minutes. (8)

(ii) State and prove Renewal equation. (8)

14. (a) (i) The density function of the time to failure (in years) of a component manufactured by a certain company is given by $f(t)=\dfrac{200}{(t+10)^3}, t\geq 0$.

(1) Derive the reliability function and determine the reliability for the first year of operation.

(2) Compute the MTTF.

(3) What is the design life for the reliability of 0.95? (8)

(ii) The time to repair a power generator is denoted by its pdf

$m(t)=\dfrac{t^2}{333}, 1\leq t\leq 10$ hours.

(1) Find the probability that the repair will be completed in 6 hours.

(2) What is the MTTR?

(3) Find the repair rate. (8)

Or

(b) (i) Given that $R(t) = e^{-\sqrt{0.001t}}$, $t \geq 0$,

(1) Compute the reliability for a 50 hour mission

(2) Find the hazard rate function

(3) Given a 10 hour warranty period, compute the reliability for a 50 hour mission.

(4) What is the average design life for a reliability of 0.95, given a 10 hour warranty period? (8)

(ii) A critical communication relay has a constant failure rate of 0.1 per day. Once it has failed the mean time to repair is 2.5 days. What are the point availability at the end of 2 days, the interval availability over a 2 day period and the steady state availability? (8)

15. (a) (i) In the railway marshalling yard, goods trains arrive at a rate of 30 trains per day. Assume that the inter-arrival time follows exponential distribution and the service time distribution is also exponential with an average 36 minutes. Calculate the following :

(1) the mean queue size

(2) the probability that the queue size exceeds 10

If the input of trains increases to an average of 33 per day, what will be the change in the above quantities? (8)

(ii) A petrol pump has 4 pumps. The service time follows an exponential distribution with a mean of 6 minutes and cars arrive for service in a Poisson process at the rate of 30 cars per hour.

(1) What is the probability that an arrival will have to wait in line?

(2) Find the average waiting time in the queue, average time spent in the system and the average number of cars in the system. (8)

Or

(b) (i) There are three typists in an office. Each typist type an average of 6 letters per hour. If letters arrive for being typed at the rate of 15 letters per hour.

(1) What fraction of time all the typist will be busy?

(2) What is the average number of letters waiting to be typed?

(3) What is the average time a letter has to spend waiting and for being typed? (8)

(ii) A 2-person barber shop has 5 chairs to accommodate the waiting customers. Potential customers who arrive when all 5 chairs are full, leave without entering the barber shop. Customers arrive at the average rate of 4 per hour and spend an average of 12 min in the barber's chair. Compute P_0, P_1, P_7 and L_q. (8)
