

H 1403

B.F./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2006.

Second Semester

MA 132 --- MATHEMATICS -- II

(Common to all branches except Information Technology)

Time : Three hours

Maximum : 100 marks

Use of Statistical Tables permitted.

Answer ALL questions.

PART A --- (10 × 2 = 20 marks)

1. Sketch roughly the region of integration for $\int_0^1 \int_0^x f(x, y) dy dx$.
2. Prove that $\frac{\beta(m+1, n)}{m} = \frac{\beta(m, n)}{m+n}$.
3. Find a such that $(3x - 2y + z)\hat{i} + (4x + ay - z)\hat{j} + (x - y + 2z)\hat{k}$ is solenoidal.
4. If $\vec{F} = x^2\hat{i} + xy\hat{j}$, evaluate $\int_C \vec{F} \cdot d\vec{r}$ along the straight line $y = x$ from (0,0) to (1,1).
5. Define analytic function of a complex variable.
6. Define conformal mapping.
7. State Taylor's theorem.
8. Define isolated singularity.

9. Given $r = 0.48$, $\text{cov}(x, y) = 36$, $\sigma_x^2 = 16$. Then find σ_y .
10. Write down Z -test significance single mean formula for a given sample and population means \bar{x} and μ respectively and the corresponding S.D.s for n variates.

PART B --- (5 × 16 = 80 marks)

11. (i) If $f(z)$ is a regular function of z , prove that $\nabla^2 |f(z)|^2 = 4 |f'(z)|^2$. (8)
- (ii) Find the bilinear transformation that maps the points $0, 1, \infty$ of the z plane into $i, 1, -i$ of the w -plane. (8)
12. (a) (i) Change the order of integration in $\int_0^{2\sqrt{x}} \int_{\frac{x^2}{4}}^{x^2} dy dx$ and then evaluate it. (8)
- (ii) Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$ by triple integration. (8)

Or

- (b) (i) Show that $\int_0^{\infty} x^n e^{-h^2 x^2} dx = \frac{1}{2h^{n+1}} \sqrt{\frac{n+1}{2}}$. Deduce that $\int_0^{\infty} e^{-h^2 x^2} dx = \frac{\sqrt{\pi}}{2h}$. Hence show that $\int_0^{\infty} \cos x^2 dx = \int_0^{\infty} \sin x^2 dx = \frac{\sqrt{\pi}}{2\sqrt{2}}$. (8)
- (ii) Evaluate $\iint r^3 dr d\theta$ over the area bounded between the circles $r = 2 \cos \theta$ and $r = 4 \cos \theta$. (8)
13. (a) (i) Find a and b such that the surfaces $ax^2 - byz = (a+2)x$ and $4x^2y + z^3 = 4$ cut orthogonally at $(1, -1, 2)$. (8)
- (ii) Verify Stoke's theorem for $\vec{F} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$ where S is the upper half of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. (8)

Or

(b) (i) Prove that $\text{curl}(\text{curl } \vec{F}) = \text{grad}(\text{div } \vec{F}) - \nabla^2 \vec{F}$. (3)

(ii) Verify Gauss divergence theorem for $\vec{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ taken over the cube bounded by $x=0, x=1, y=0, y=1, z=0$ and $z=1$. (3)

14. (a) (i) Using Cauchy's integral formula, evaluate $\int_C \frac{z dz}{(z-1)(z-2)^2}$ where

C is the circle $|z-2| = \frac{1}{2}$. (3)

(ii) Using the method of contour integration, evaluate

$$\int_0^{2\pi} \frac{d\theta}{1 - 2x \cos \theta + x^2}, \quad 0 < x < 1. \quad (8)$$

Or

(b) (i) Find the Laurent's series expansion of $f(z) = \frac{1}{z-z^2}$ in the region $1 < |z+1| < 2$ and $|z+1| > 2$. (8)

(ii) Evaluate $\int_0^8 \frac{dx}{(1+x^2)^2}$ using contour integration. (8)

15. (a) (i) Compute α_3 and β_2 for the following distribution.

Class x : 3-7 8-12 13-17 18-22 23-27 28-32 33-37 38-42

Frequency f : 2 108 580 175 80 32 18 5

(8)

(ii) Samples of two types of electric bulbs were tested for length of life and the following data were obtained.

	Size	Mean	S.D.
Sample A	8	1234 hrs	36 hrs
Sample B	7	1036 hrs	40 hrs

Is the difference in the means sufficient to warrant that type A bulbs are superior to type B bulbs? (8)

Or

- (b) (i) A die is thrown 498 times. Denoting x to be the number appearing on the top face of it, the observed frequency of x is given below :

x :	1	2	3	4	5	6
f :	69	78	85	82	86	98

What opinion you would form for the accuracy of the die? (8)

- (ii) Find the coefficient of correlation between the marks of Mathematics and Physics of 9 students given below.

Marks in Mathematics : 72 65 70 71 69 67 65 67 66

Marks in Physics : 69 70 70 70 70 68 67 69 68

(8)